Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2015

Assignment #S(3)
Due on Tuesday, 13 October, 2015.

Induction

Recall from class that we defined $+$ and $\cdot$ on $\mathbb{N}$ by induction from the *successor function*, $S(n) = n \cup \{n\}$ (informally, of course, $S(n) = n + 1$) as follows:

- For all $n \in \mathbb{N}$, let $n + 0 = n$.
- For all n, $k \in \mathbb{N}$, given that $n + k$ has been defined, let $n + S(k) = S(n + k)$.

and, given that $+$ has been defined,

- For all $n \in \mathbb{N}$, let $n \cdot 0 = 0$.
- For all n, $k \in \mathbb{N}$, given that $n \cdot k$ has been defined, let $n \cdot S(k) = (n \cdot k) + n$.

We showed in class that $+$ was commutative, *i.e.* $n + k = k + n$ for all n, $k \in \mathbb{N}$, using lots of induction.

1. Use induction to show that $\cdot$ is associative on $\mathbb{N}$, *i.e.* $(n \cdot k) \cdot m = n \cdot (k \cdot m)$ for all n, k, $m \in \mathbb{N}$. You may assume all the usual algebraic properties of $+$, as well as the commutativity of $\cdot$ and the fact that $n \cdot 1 = n$ for all $n \in \mathbb{N}$. *[10]*