Mathematics 2200H – Mathematical Reasoning
Trent University, Fall 2015

Assignment #1 – *Due on Monday. 21 September.*

A Serpent With Corners

"Water, water, every where,
Nor any drop to drink."

"It'll take just one more pebble."

"What ever *are* you doing with those buckets?"

The speakers were Hugh and Lambert. Place, the beach of Little Mendip. Time 1.30, p.m. Hugh was floating a bucket in another a size larger, and trying how many pebbles it would carry without sinking. Lambert was lying on his back, doing nothing.

For the next minute or so Hugh was silent, evidently deep in thought. Suddenly he started. "I say, look here, Lambert!" he cried.

"If it's alive, and slimy, and with legs, I don't care to," said Lambert.

"Didn't Balbus say this morning that, if a body is immersed in liquid, it displaces as much liquid as is equal to its own bulk?" said Hugh.

"He said things of that sort," Lambert vaguely replied.

"Well, just look here a minute. Here's the little bucket almost immersed: so the water displaced ought to be just about the same bulk. And now just look at it!" He took out the little bucket as he spoke, and handed the big one to Lambert. "Why, there's hardly a teacupful! Do you mean to say *that* water is the same bulk as the little bucket?"

"Course it is," said Lambert.

"Well, look here again!" cried Hugh, triumphantly, as he poured the water from the big bucket into the little one. "Why, it doesn't half fill it!"

"That's *its* business," said Lambert. "If Balbus says it's the same bulk, why, it *is* the same bulk, you know."

"Well, I don't believe it," said Hugh.

"You needn't," said Lambert. "Besides, it's dinner-time. Come along."

They found Balbus waiting dinner for them, and to him Hugh at once propounded his difficulty.

"Let's get you helped first," said Balbus, briskly cutting away at the joint. "You know the old proverb 'Mutton first, mechanics afterwards'?"

"The boys did *not* know the proverb, but they accepted it in perfect good faith, as they did every piece of information, however startling, that came from so infallible an authority as their tutor. They ate on steadily in silence, and, when dinner was over, Hugh set out the usuall array of pens , ink, and paper, while Balbus repeated to them the problem he had prepared for their afternoon's task.

"A friend of mine has a flower-garden—a very pretty one, though of no great size—"

"How big is it?" said Hugh.

"That's what *you* have to find out!" Balbus gaily replied. "All *I* tell you is that it is oblong in shape—just half a yard longer than its width—and that a gravel-walk, one yard wide, begins at one corner and runs all around it."

"*Not* joining into itself, young man. Just before doing *that*, it turns a corner, and runs round the garden again, alongside of the first portion, and then inside that again, winding in and in, and each lap touching the last one, till it has used up the whole of the area."

"Like a serpent with corners?" said Lambert.

"Exactly so. And if you walk the whole length of it, to the last inch, keeping in the centre of the path, it's exactly two miles and half a furlong. Now, while you find out the length and breadth of the garden, I'll see if I can think out that sea-water puzzle."

"You said it was a flower-garden?" Hugh inquired, as Balbus was leaving the room.

"I did," said Balbus.

"Where do the flowers grow?" said Hugh. But Balbus thought it best not to hear the question. He left the boys to their problem, and, in the silence of his own room, set himself to unravel Hugh's mechanical paradox.

"To fix our thoughts," he murmured to himself, as, with hands deep-buried in his pockets, he paced up and down the room, "we will take a cylindrical glass jar, with a scale of inches marked up the side, and fill it with water up to the 10-inch mark: and we will assume that every inch depth of jar contains a pint of water. We will now take a solid cylinder, such that every inch of it is equal in bulk to *half* a pint of water, and plunge 4 inches of it into the water, so that the end of the cylinder comes down to the 6-inch mark. Well, that displaces 2 pints of water. What becomes of them? Why, if there were no more cylinder, they would lie comfortably on the top, and fill the jar up to the 12-inch mark. But unfortunately there *is* more cylinder, occupying half the space between the 10-inch and the 12-inch marks, so that only *one* pint of water can be accommodated there. What becomes of the other pint? Why, if there were no more cylinder, it would lie on the top, and fill the jar up to the 13-inch mark. But unfortunately——Shade of Newton!" he exclaimed, in sudden accents of terror. "When *does* the water stop rising?"

A bright idea struck him. "I'll write a little essay on it," he said.

Balbus's Essay

"When a solid is immersed in a liquid, it is well known that it displaces a portion of the liquid equal to itself in bulk, and that the level of the liquid rises just so much as it would rise if a quantity of the liquid had been added to it, equal in bulk to the solid. Lardner says, precisely the same process occurs when a solid is *partially* immersed: the quantity of the liquid displaced, in this case, equalling the portion of the solid which is immersed, and the rise of the level being in proportion.

"Suppose a solid held above the surface of a liquid and partially immersed: a portion of the liquid is displaced, and the level of the liquid rises. But, by this rise of level, a little bit more of the solid is of course immersed, and so there is a new displacement of a second portion of the liquid, and a consequent rise of level. Again, this second rise of level causes a yet further immersion, and by consequence another displacement of liquid and another rise. It is self-evident that this process must continue till the entire solid is immersed, and that the liquid will then begin to immerse whatever holds the solid, which, being connected with it, must for the time be considered a part of it. If you hold a stick, six feet long, with its end in a tumbler of water, and wait long enough, you must eventually be immersed. The question as to the source from which the water is supplied—which belongs to a high branch of mathematics, and is therefore beyond our present scope—does not apply to the sea. Let us therefore take the familiar instance of a man standing at the edge of the sea, at ebb-tide, with a solid in his hand, which he partially immerses: he remains steadfast and unmoved, and we all know that he must be drowned. The multitudes who daily perish in this manner to attest to a philosophical truth, and whose bodies the unreasoning wave casts sullenly upon our thankless shores, have a truer claim to be called the martyrs of science than a Galileo or a Kepler. To use Kossuth's eloquent phrase, they are the unnamed demigods of the nineteenth century."*

"There's a fallacy *somewhere*," he murmured drowsily, as he stretched his long legs upon the sofa. "I must think it over again." He closed his eyes, in order to concentrate his attention more perfectly, and for the next hour or so his slow and regular breathing bore witness to the careful deliberation with which he was investigating this new and perplexing view of the subject.

1. Identify, analyze, and solve all the mathematical problems posed in the story above. *[10]*

2. Do each of the following from Chapter 1 of the textbook: Exercises 2, 8, 12, 14, & 18
[10 = 5 × 2 each]

* *Note by the writer.*—For the above Essay I am indebted to a dear friend, now deceased.