Mathematics 2084H – Recreational mathematics
TRENT UNIVERSITY, Winter 2009

Assignment #2
Due on Friday, 30 January, 2009.

Magic Squares

See pages 202-210 in the text for a brief discussion of symmetric and pandiagonal magic squares. In this assignment we will only consider *standard* $n \times n$ magic squares, namely those using all the integers from 1 through n^2.

Symmetric magic squares. Let a_{ij} denote the entry in row i and column j of an $n \times n$ magic square. Recall that a magic square using all the integers from 1 through n^2 is said to be *symmetric* (or *associated*) if $a_{ij} + a_{st} = n^2 + 1$ whenever a_{ij} and a_{st} are in symmetric positions relative to the centre of the square (*i.e.* whenever $i + s = j + t = n + 1$). The methods given in class for constructing magic squares for odd n and for n a multiple of 4 both give symmetric magic squares.

1. Suppose we have an $n \times n$ symmetric magic square. Explain why it remains a magic square if one swaps rows or columns in symmetric positions (*i.e.* rows or columns k and $n - k + 1$ for some k). *[3]*

2. Give an example to demonstrate that such a swap applied to a magic square that is not symmetric need not result in a magic square. *[1]*

3. Explain why the following procedure, when applied to any magic square, symmetric or not, gives a magic square: Swap the first and last columns and then swap the first and last rows. *[2]*

Pandiagonal magic squares. Consider the following 5×5 magic square:

1	15	24	8	17
23	7	16	5	14
20	4	13	22	6
12	21	10	19	3
9	18	2	11	25

A broken diagonal in a magic square is one obtained by wrapping around the edges. For example, 23 15 2 19 6 is a broken diagonal of the given square, as is 17 23 4 10 11. This particular square is *pandiagonal* (also called *panmagic* or *nasik*): it has the property that all of its broken diagonals also add up to the magic constant. (This example is also symmetric.) The methods given in class for constructing magic squares will not usually result in a pandiagonal magic square.

4. Explain why there can be no 2×2, 3×3, or 4×4 pandiagonal magic square using all the integers from 1 through n^2 (for $n = 2$, 3, or 4, respectively, of course). *[4]*