

**MATH 1550**

**Fall 2023**

**Test 2**

**2023-3-15**

**Time Limit: 50 minutes**

**Name (Print):** \_\_\_\_\_

**ID number:** \_\_\_\_\_

---

### **Instructions**

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator, so long as it is not capable of wirelessly communicating with other devices or accessing the internet.
- A single leaf, double-sided, A4 size, aid sheet is allowed.

1. (5 points) Do any 2 of the following for 2.5 points each:

(a) Suppose  $F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x \leq 1 \\ 1 & x > 0 \end{cases}$  is the cumulative distribution function of the continuous random variable  $X$ . What is the probability density function of  $X$ ?

(b) A fair four-sided die has faces numbered 1, 2, 3, and 5. What is the expected value of the number that comes up when the die is rolled once?

(c) If  $f(t) = \begin{cases} 1+t & t \in [-1, 0] \\ 1-t & t \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$  is the probability density function of the continuous random variable  $T$ . Find  $P(-0.5 \leq T \leq 0.5)$ .

2. (5 points) Do any 2 of the following for 2.5 points each:

(a) Determine whether  $f(x, y) = \begin{cases} x^2y^2 & x \in [-1, 1] \text{ and } y \in [-1, 1] \\ 0 & \text{otherwise} \end{cases}$ , is a valid joint probability density function.

(b) The joint probability distribution function  $f(x, y)$  of the discrete random variables  $X$  and  $Y$  is given by the table below. Find  $P(X = 1|Y = 5)$ .

		$x$		
		0	1	2
$y$	3	0.1	0	0.1
	4	0.1	0.1	0.2
	5	0.2	0.1	0.1

(c) Two fair six-sided dice are given, one blue and one red. The blue one has faces numbered 0 through 5, and the red one has faces numbered 2 through 7. One of these dice is selected at random (with equal probability) and then rolled once. The random variable  $X$  counts the number of times the blue die is chosen, and the random variable  $Y$  gives the number on the face that comes up when the selected die is rolled. Give the complete table describing the joint distribution function of  $X$  and  $Y$ .

3. (5 points) Do any 1 of the following for 5 points:

(a) Let  $f(x) = \begin{cases} 4(x^3 - x) & -1 \leq x \leq 0 \\ 0 & \text{otherwise} \end{cases}$ , be the probability density function of the continuous random variable  $X$ . Find the expected value,  $E(X)$ , of  $X$ .

(b) The joint probability distribution function  $f(x, y)$  of the discrete random variables  $X$  and  $Y$  is given by the table below. Find the expected value,  $E(Y - X)$ , of  $Y - X$ .

		$x$		
		0	1	2
$y$	3	0.1	0	0.1
	4	0.1	0.1	0.2
	5	0.2	0.1	0.1

BonusBonusBonus

4. (1 Bonus points) One hundred people line up to board an airplane. Each has a boarding pass with assigned seat. However, the first person to board has lost their boarding pass and takes a random seat. After that, each person takes the assigned seat if it is unoccupied, and one of the unoccupied seats at random otherwise. What is the probability that the last person to board gets to sit in their assigned seat?