

1. The joint probability distribution for discrete random variables X and Y is given in the table below.

		x		
		0	1	2
y	0	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
	1	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

- (a) Verify that this is a valid joint probability distribution.
 - (b) Find $P(Y = 0, X \leq 1)$.
 - (c) Find the marginal distributions for X and Y .
 - (d) Find the conditional probability distributions $P(X = x|Y = 0)$ and $P(Y = y|X = 1)$.
 - (e) Find the conditional probabilities
 - i. $P(X = 1|Y = 0)$
 - ii. $P(X = 2|Y = 1)$
 - iii. $P(Y = 1|X = 1)$
 - iv. $P(Y = 0|X = 2)$.
 - (f) Are X and Y independent?
2. A bag contains 40 blue marbles and 60 red marbles. Suppose 10 marbles are drawn from the bag without replacement. Let X be the number of blue marbles drawn, and Y the number of red marbles drawn.
- (a) Give the joint probability distribution for X and Y .
 - (b) Find the marginal distributions for X and Y .
 - (c) Find the conditional probability distributions $P(Y = y|X = 0)$ and $P(X = x|Y = 3)$.
 - (d) Are X and Y independent?
3. Let X and Y be joint continuous random variables with joint probability density function given below.

$$f(x, y) = \begin{cases} x + Cy^2 & - \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{else} \end{cases}$$

- (a) Determine an appropriate value for $C \in \mathbb{R}$ (if one exists).
- (b) Find $P(0 \leq X \leq \frac{1}{2}, \frac{1}{2} \leq Y \leq 1)$.
- (c) Find the joint cumulative distribution function $F(x, y)$ for X and Y .
- (d) Find the marginal distributions for X and Y .
- (e) Find the conditional probability distributions $P(Y = y|X = 0)$ and $P(X = x|Y = 0.5)$.
- (f) Are X and Y independent?