MATH1550, Winter 2023 **Names:**
Seminar 6

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1. Find the cumulative distribution function $F(x)$ for each probability density function $f(x)$.

   (a)
   $$f(x) = \begin{cases} \frac{3}{8}(4x - 2x^2) & \text{for } x \in [0, 2] \\ 0 & \text{else} \end{cases}$$

   (b)
   $$f(x) = \begin{cases} \frac{10}{x^2} & \text{for } x > 10 \\ 0 & \text{else} \end{cases}$$

   (c)
   $$f(x) = \begin{cases} 10(x^3 - x^4) & \text{for } x \in [0, 1] \\ \frac{4}{3x^3} & \text{for } x \in (1, 2] \\ 0 & \text{else} \end{cases}$$

2. The cumulative distribution function $F(x)$ for a continuous random variable X is given. Find the probability density $f(x)$ for X.

   (a)
   $$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ x^2 & \text{for } 0 \leq x < 1 \\ 1 & \text{for } x \geq 1 \end{cases}$$

   (b)
   $$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{x^2}{2} & \text{for } 0 \leq x \leq 1 \\ 2x - \frac{x^2}{2} - 1 & \text{for } 1 < x \leq 2 \\ 1 & \text{for } x > 2 \end{cases}$$

   (c)
   $$F(x) = \begin{cases} 1 - e^{-x} & \text{for } x \geq 0 \\ 0 & \text{else} \end{cases}$$

3. A fair coin is tossed 4 times. You win \$3 if 2 or 4 heads appear, you win \$1 if 1 or 3 heads appear and you lose \$6 if if no heads appear. Let X be the number of heads, and Y the number of dollars won, after 4 tosses. Give the joint probability distribution $f(x, y)$, for X and Y.

4. Two fair 6-sided dice are thrown. Let X be the largest value appearing on either die, and Y be value appearing on the first die. Give the joint probability distribution $f(x, y)$, for X and Y.

5. A fair coin is tossed three times. Let X be the number of heads that appear, and Y the toss (1, 2 or 3) where heads first appears, or $Y = 0$ if heads dose not appear. Give the joint probability distribution $f(x, y)$, for X and Y.

6. The joint probability distribution for discrete random variables X and Y is given in the table below.

		x		
		1	2	3
y	0	$\frac{1}{8}$	k	$\frac{1}{8}$
	2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{6}$
	4	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$

(a) Determine an appropriate value for $k \in \mathbb{R}$.

(b) Find $P(X = 1, Y = 4)$.

(c) Find $P(X \leq 2.25, Y \leq 3)$.

(d) Find $P(X \leq 2.6, Y > 1)$.