MATH1550, Winter 2023 **Names:**
Seminar 5

---

1. A continuous random variable X is said to have *uniform density* (or *equiprobable density*) if its probability density function is constant on some region and zero elsewhere; for example if $C \in \mathbb{R}$, then

$$f(x) = \begin{cases} C & \text{for } x \in [a,b] \\ 0 & \text{else} \end{cases}$$

is such a function. The set $[a,b]$, where f is nonzero, is called the *support* of f. This type of probability density function models the situation of having an equally likely chance of choosing a number at random from the interval $[a,b]$.

   (a) Let X be a number chosen at random from the interval $[3,15]$. Give the probability density function for X assuming it has uniform density.

   (b) Use the probability density function from part (a) to compute:
      i. $P(7 \leq X \leq 10)$
      ii. $(X < 10)$
      iii. $P(0 \leq X \leq 20)$
      iv. $P(1 \leq X < 4)$
      v. $P(X^2 - 2X - 9 < 0)$

   (c) What must the value for C be for the uniform density function f with support $[a,b]$?

   (d) In this case, give a formula for finding $P(c \leq X \leq d)$ where $a \leq c \leq d \leq b$.

2. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{C}{x^3} & \text{for } x \in [a,b] \\ 0 & \text{else} \end{cases}$$

where $C \in \mathbb{R}$ and $a < b$.

   (a) Find an appropriate value for C if $a = 1$ and $b = 2$.

   (b) Find an appropriate value for C for arbitrary a and b with $0 < a < b$.

3. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} C(4x - 2x^2) & \text{for } x \in [0,2] \\ 0 & \text{else} \end{cases}$$

   (a) Show that $C = \frac{3}{8}$.

   (b) Compute $P(X > 1)$.

4. The lifetime (in years) of a certain machine component is a random variable with probability density function

$$f(x) = \begin{cases} \frac{10}{x^2} & \text{for } x > 10 \\ 0 & \text{else} \end{cases}$$

   (a) What is the probability that such a component will last more than 20 years?.

   (b) A machine contains 6 such components. The lifetime of any component is unaffected by the others. What is the probability that exactly 4 of them last over 15 years? Hint: Let A_i be the event that component i will last more then 15 years, and assume that events $A_1, A_2, A_3, A_4, A_5, A_6$ are independent.

5. You arrive at a bus stop at 10:00 AM knowing that the bus will arrive some time between 10:00 AM and 10:30 AM with equal likelihood.

   (a) What is the probability that you will have to wait longer than 10 minutes for the bus to arrive?

   (b) If at 10:15 AM the bus has not arrived, what is the probability that you will have to wait at least 10 more minutes?

6. Determine whether or not the following function may serve as a valid probability density function.

$$f(x) = \begin{cases} \frac{4}{3}(2x - x^3) & \text{for } \frac{1}{2} \leq x \leq \frac{3}{2} \\ 0 & \text{else} \end{cases}$$