MATH1550, Winter 2023 **Names:**
Seminar 3

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1. Two fair 6-sided dice are thrown.

   (a) What is the conditional probability that at least one die lands on 6 given that the dice land on different numbers?

   (b) What is the conditional probability that the first die lands on 6 given that the sum of the dice is x? Compute for all values of $x \in \{2, \dots, 12\}$.

   Solution.

   (a) Let A be the events that one die lands on a 6, and B be the event that the dice land on different numbers. Then

   $$A = \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6),(5,6),(4,6),(3,6),(2,6),(1,6)\}.$$

   We see that $|B| = 6 \cdot 5 = 30$ and

   $$A \cap B = \{(6,1),(6,2),(6,3),(6,4),(6,5),(5,6),(4,6),(3,6),(2,6),(1,6)\}.$$

   Thus

   $$P = (A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{10}{36}}{\frac{30}{36}} = \frac{1}{3}$$

   (b) Let C be the event that the first die lands on 6, and S_x the event that the sum of the dice is x. Then

   $$C = \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\},$$

x	2	3	4	5	6	7	8	9	10	11	12
$P(S_x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

   and for each $x \in \{2, \dots, 12\}$,

   $$P(C|S_x) = \frac{P(C \cap S_x)}{P(S_x)}.$$

   We see that

   $$C \cap S_x = \begin{cases} \emptyset & \text{for } x = 2,3,4,5,6 \\ \{(6, x-6) & \text{for } x = 7,8,9,10,11,12 \end{cases}$$

   Thus

x	2	3	4	5	6	7	8	9	10	11	12
$P(C|S_x)$	0	0	0	0	0	1/6	1/5	1/4	1/3	1/2	1

   □

2. In a certain city, 46 percent of the voters classify themselves as Liberal supporters, whereas 30 percent support Conservatives, and 24 percent support the NDP. In a recent election, 35 percent of the Liberal, 62 percent of the Conservative, and 58 percent of the NDP supporters came out to vote. A voter is chosen at random.

   (a) Given that this person did vote what is the probability that they support

      i. Liberal,

      ii. Conservative,

      iii. NPD?

   (b) What percentage of voters participated in the election?

Solution. Let V be the set of people who voted, and L, C, NDP the set of Liberal, Conservative and NPD supporters respectively. By the rule of total probability

$$\begin{aligned} P(V) &= P(V|L)P(L) + P(V|C)P(C) + P(V|NPD)P(NPD) \\ &= (0.35)(0.46) + (0.62)(0.3) + (0.58)(0.24) \\ &= 0.4862 \end{aligned}$$

(a) Given that this person did vote what is the probability that they support

i.
$$P(L|V) = \frac{P(L \cap V)}{P(V)} = \frac{P(V|L)P(L)}{P(V)} = \frac{(0.35)(0.46)}{0.4862} = \frac{805}{2431} \approx 0.3311$$

ii.
$$P(C|V) = \frac{P(C \cap V)}{P(V)} = \frac{P(V|C)P(C)}{P(V)} = \frac{(0.62)(0.3)}{0.4862} = \frac{930}{2431} \approx 0.3826$$

iii.
$$P(NDP|V) = \frac{P(NDP \cap V)}{P(V)} = \frac{P(V|NDP)P(NDP)}{P(V)} = \frac{(0.58)(0.24)}{0.4862} = \frac{696}{2431} \approx 0.2863$$

(b) As seen above $P(V) = 0.4862$.

□

3. Two cards are randomly chosen without replacement from regular 52-card deck. Let A be the event that at least one ace is chosen, A_S the event that the aces of spades is chosen, and B the event that both cards are aces. Find

(a) $P(B|A_S)$

(b) $P(B|A)$

Solution. (a) By definition,

$$P(B|A_S) = \frac{P(B \cap A_S)}{P(A_S)} = \frac{\frac{\binom{3}{1}}{\binom{51}{2}}}{\frac{\binom{51}{1}}{\binom{52}{2}}} = \frac{1}{17}$$

of course we can see this more directly as,

$$P(B|A_S) = \frac{(\text{\# of 2-ace hands with one of them being the ace of spades})}{(\text{\# of 2-card hands with one card being the ace of spaces})} = \frac{3}{51} = \frac{1}{17}.$$

(b)

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)}{P(A)} = \frac{\frac{\binom{4}{2}}{\binom{52}{2}}}{\frac{\binom{4}{1}\binom{51}{1}}{\binom{52}{2}}} = \frac{1}{34}.$$

□

4. Show that if events A and B are independent, then events A' and B are independent. Hint: $A' \cap B = B \setminus A$.

Solution. If A and B and are independent then $P(A \cap B) = P(A)P(B)$. Now

$$P(A' \cap B) = P(B \setminus A) = P(B) - P(A \cap B) = P(B) - P(A)P(B) = P(B)(1 - P(A)) = P(B)P(A'),$$

therefore A' and B are independent.

$\square$

5. An explosion at a construction site could have occurred as the result of static electricity, malfunctioning equipment, carelessness, or sabotage. Research estimates that such an explosion would occur with probability of 0.25 as a result of static electricity, 0.20 as a result of malfunctioning equipment, 0.40 as a result of carelessness, and 0.75 as a result of sabotage. The four causes for explosion occur with probabilities 0.20, 0.40, 0.25 and 0.15 respectively.

   (a) What is the most likely cause of the explosion?

   (b) What is the least likely cause of the explosion?

   Solution. Let E be the event that an explosion occurs, Sta, Mal, Car, and Sab be the events of static electricity, malfunctioning equipment, carelessness, or sabotage respectively. We are given the following:

$$P(Sta) = 0.2, P(Mal) = 0.4, P(Car) = 0.25, P(Sab) = 0.15,$$

$$P(E|Sta) = 0.25, P(E|Mal) = 0.2, P(E|Car) = 0.4, P(E|Sab) = 0.75.$$

   Then

$$\begin{aligned} P(E) &= P(E|Sta)P(Sta) + P(E|Mal)P(Mal) + P(E|Car)P(Car) + P(E|Sab)P(Sab) \\ &= (0.25)(0.2) + (0.2)(0.4) + (0.4)(0.25) + (0.75)(0.15) \\ &= 0.3425 \end{aligned}$$

   and

$$\begin{aligned} P(Sta|E) &= \frac{P(Sta)P(E|Sta)}{P(E)} = \frac{(0.2)((0.25)}{0.3425} = \frac{20}{137} \approx 0.1460 \\ P(Mal|E) &= \frac{P(Mal)P(E|Mal)}{P(E)} = \frac{(0.4)((0.2)}{0.3425} = \frac{32}{137} \approx 0.2336 \\ P(Car|E) &= \frac{P(Car)P(E|Car)}{P(E)} = \frac{(0.25)((0.4)}{0.3425} = \frac{40}{137} \approx 0.2920 \\ P(Sab|E) &= \frac{P(Sab)P(E|Sab)}{P(E)} = \frac{(0.15)((0.75)}{0.3425} = \frac{45}{137} \approx 0.3285 \end{aligned}$$

   (a) The most likely cause of explosion is sabotage.

   (b) The least likely cause of explosion is static electricity.

$\square$

6. There are 90 applicants for a job with a local television station. Some of them are university graduates and some are not. Some of them have at least 3 years of experience ans some don't. This information is summarized in the table below.

	Univ. Grad.	Not a Univ. Grad.
≥ 3 years exp.	18	9
< 3 years exp.	36	27

   Let G be the event that an applicant is a university graduate, and T the event that an applicant has at least 3 years of experience. Find:

   (a) $P(G)$

   (b) $P(T')$

(c) $P(G \cap T)$

(d) $P(G' \cap T')$

(e) $P(T|G)$

(f) $P(G'|T')$

(g) Verify that $P(T|G) = \frac{P(G \cap T)}{P(G)}$

Solution.

(a)
$$P(G) = \frac{18 + 36}{90} = \frac{54}{90} = \frac{3}{5} = 0.6$$

(b)
$$P(T') = \frac{36 + 27}{90} = \frac{63}{90} = \frac{7}{10} = 0.7$$

(c)
$$P(G \cap T) = \frac{18}{90} = \frac{1}{5} = 0.2$$

(d)
$$P(G' \cap T') = \frac{27}{90} = \frac{3}{10} = 0.3$$

(e)
$$P(T|G) = \frac{18}{18 + 36} = \frac{18}{54} = \frac{1}{3} \approx 0.3333$$

(f)
$$P(G'|T') = \frac{27}{36 + 27} = \frac{27}{63} = \frac{3}{7} \approx 0.4286$$

(g)
$$\frac{P(G \cap T)}{P(G)} = \frac{\frac{18}{90}}{\frac{27}{90}} = \frac{1}{3} = P(T|G)$$

□