MATH1550, Winter 2023 **Names:**
Seminar 2

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1. A bag contains 6 marbles of which 3 are red, 2 are green and 1 is blue. Consider the experiment of drawing 1 marble from the bag (without looking), returning the marble to bag, and then drawing 1 marble again. (Here we say the marbles are drawn *with replacement*).

   (a) Write the sample space for this experiment as described.

   (b) Suppose that the first marble is not returned to the bag. Write the sample space in this case. (Here we say the marbles are drawn *without replacement*).

   (c) According the the classical probability concept, what is the probability that both marbles drawn are red when they are drawn with replacement?

   (d) What is the probability that both marbles drawn are red when they are drawn without replacement?

Solution.

(a)
$$S_1 = \{(R,R),(R,G),(R,B),(G,R),(G,G),(G,B),(B,R),(B,G),(B,B)\}$$

(b)
$$S_2 = \{(R,R),(R,G),(R,B),(G,R),(G,G),(G,B),(B,R),(B,G)\}$$

(c) Note that the outcomes in S_1 are not equally likely. The number of ways of getting (R,R), when drawing with replacement, is
$$n = \binom{3}{1}\cdot\binom{3}{1} = 9.$$
The total number of ways that any 2 marbles can be drawn with replacement is
$$N = \binom{6}{1}\cdot\binom{6}{1} = 36.$$
Therefore, the probability that both marbles drawn are red is
$$\frac{n}{N} = \frac{9}{36} = \frac{1}{4}.$$

(d) Again, the outcomes in S_2 are not equally likely. The number of ways of getting (R,R), when drawing without replacement, is
$$n = \binom{3}{1}\cdot\binom{2}{1} = 6.$$
The total number of ways that any 2 marbles can be drawn without replacement is
$$N = \binom{6}{1}\cdot\binom{5}{1} = 30.$$
Therefore, the probability that both marbles drawn are red is
$$\frac{n}{N} = \frac{6}{30} = \frac{1}{5}.$$

□

2. Alex, Blake and Charley take turns flipping a coin (starting with Alex, then Blake, then Charley). The first person to get heads wins. Let $A/B/C$ be the events that Alex/Blake/Charley wins respectively.

(a) Describe the sample space S for this experiment.

(b) Write the subset A of S.

(c) Write the subset B of S.

(d) Write the subset $A \cap B$.

(e) Write the subset $A \cup C$

(f) Write the subset $(A \cup B)'$ (the complement of $A \cup B$).

Solution.

(a)
$$S = \{TTTT\ldots, H, TH, TTH, TTTH, TTTTH, TTTTTH, \ldots\}$$

(b)
$$A = \{H, TTTH, TTTTTTH, TTTTTTTTTH, \ldots\} = \{T^{3k}H | k \in \mathbb{Z}, k \geq 0\}$$

(c)
$$B = \{TH, TTTTH, TTTTTTTH, TTTTTTTTTTH, \ldots\} = \{T^{3k+1}H | k \in \mathbb{Z}, k \geq 0\}$$

(d)
$$A \cap B = \emptyset$$

(e)
$$\begin{aligned} A \cup C &= \{H, TTTH, TTTTTTH, \ldots\} \cup \{TTH, TTTTTH, TTTTTTTTH, \ldots\} \\ &= \{T^{3k}H, T^{3k+2}H | k \in \mathbb{Z}, k \geq 0\} \end{aligned}$$

(f)
$$(A \cup B)' = C \cup \{TTTT\ldots\}.$$

□

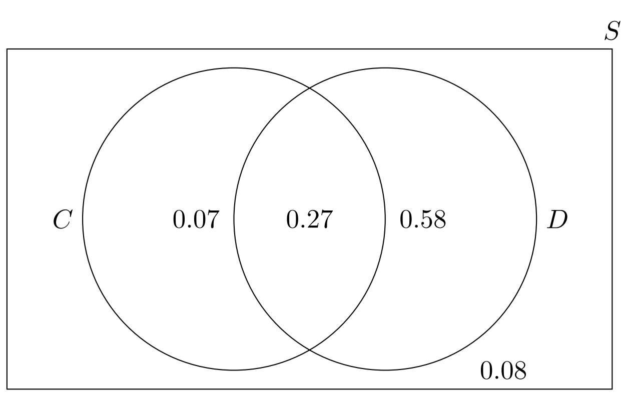
3. A certain campus restaurant only accepts payment by debit or credit. It is known that 34 percent of the students on campus carry a credit card, 85 percent of the students on campus carry a debit card and 27 percent carry both.

(a) Draw a Venn diagram representing this situation.

(b) What percentage of the students on campus will be able to make a purchase at this particular restaurant?

Solution.

(a)



(b) The percentage of students that will be able to make a purchase at this restaurant is

$$C \cup D = P(C) + P(D) - P(C \cap D) = 0.34 + 0.85 - 0.27 = 0.92.$$

□

4. Suppose a sample of Canadians are surveyed, and it is found that 22 percent speak French, 15 percent speak Spanish and 8 percent speak both French and Spanish.

   (a) What percentage speaks neither French nor Spanish?

   (b) What percentage speaks French but not Spanish?

   Solution. Let F and S be the event that a person, chosen at random from this sample, speaks French and Spanish respectively.

   (a) The percentage of people that speak neither French nor Spanish is

   $$P((F \cup S)') = 1 - P(F \cup S) = 1 - (P(F) + P(S) - P(F \cap S) = 1 - 0.22 - 0.15 + 0.08 = 0.71.$$

   (b) Note that

   $$F = (F \setminus S) \cup (F \cap S),$$

   i.e. the set of people who speak French is made up of the set of people who speak French but not Spanish and the set of people who speak both French and Spanish. Since this is a disjoint union of sets, we have

   $$P(F) = P(F \setminus S) + P(F \cap S).$$

   Rearranging we get

   $$P(F \setminus S) = P(F) - P(F \cap S) = 0.22 - 0.08 = 0.14.$$

□

5. A pair of 6-sided dice are thrown. What is the probability that the second die is larger than the first?

   Solution. Since the are 6 numbers per die, there are $N = 36$ outcomes in the sample space for this experiment. Here they are listed

$$\begin{array}{cccccc}
(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\
(2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\
(3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\
(4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\
(5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\
(6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6)
\end{array}$$

   We can see that for each $i \in \{1, 2, 3, 4, 5, 6\}$, there are $6 - i$ pairs where the second entry is larger than the first. Thus there are

   $$n = 5 + 4 + 3 + 2 + 1 + 0 = 15$$

   such pairs. Therefore the probability that the second die is larger than the first is

   $$\frac{n}{N} = \frac{15}{36} = \frac{5}{12}.$$

□

6. A magazine publishes an article on a study done with 1000 people. The study shows that of these 1000 people:

- 470 are married,
- 525 have university degrees,
- 312 have professional certifications,
- 147 are married and have university degrees,
- 42 have university degrees and professional certifications,
- 86 are married and have professional certifications, and
- 25 are married, have a university degree, and a professional certifications.

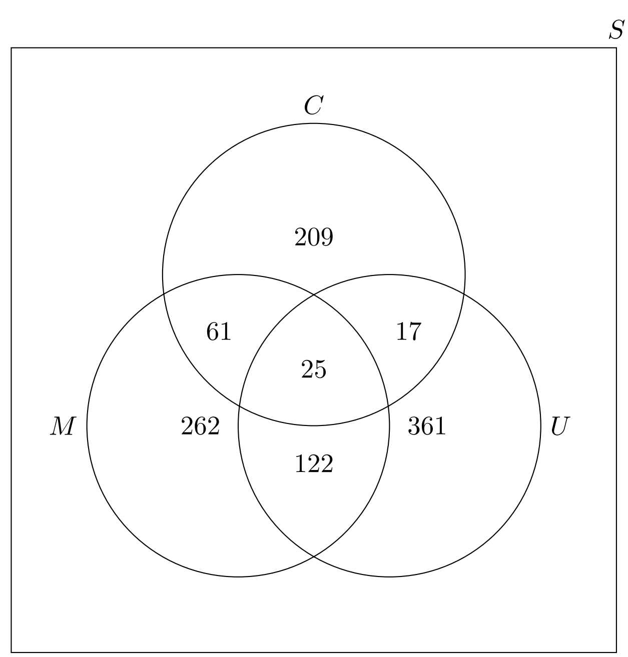
Give a reason why we should be skeptical of this study. Hint: Show that these numbers violate the rules of probability.

Solution. Let M, U, and C, denote the event that a person selected at random from the 1000 is married, has a university degree, and has a professional certificate respectively. Then

$$\begin{aligned} P(M \cup U \cup C) &= P(M) + P(U) + P(C) - P(M \cap U) - P(M \cap C) - P(U \cap C) + P(M \cup U \cap C) \\ &= \frac{470}{1000} + \frac{525}{1000} + \frac{312}{1000} - \frac{147}{1000} - \frac{42}{1000} - \frac{86}{1000} + \frac{25}{1000} \\ &= \frac{1057}{1000}. \end{aligned}$$

This violates our postulates of probability, in that no probability may be greater than 1. Therefore, some of the information in this study must be incorrect.

Below is a Venn diagram showing the breakdown of these people according to the study. Notice that there are 1057 people accounted for.



$\square$