MATH1550, Winter 2023 **Names:**
Seminar 10

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1. Suppose X and Y are discrete random variables with joint probability distribution given below.

		x = 1	2	3
y	1	0.10	0.08	0.06
	2	0.12	0.12	0.06
	3	0.16	0.10	0.20

   (a) Find $E(X)$ and $E(Y)$.

   (b) Find $\text{var}(X)$ and $\text{var}(Y)$.

   (c) Find $\text{cov}(X, Y)$.

   (d) The *correlation* between X and Y is defined as $\rho(X, Y) = \frac{\text{cov}(X,Y)}{\sqrt{\text{var}(X)}\sqrt{\text{var}(Y)}}$, provided $\text{var}(X), \text{var}(Y) \neq 0$. It provides a measure of the degree of linearity between X and Y (this would appear in course on statistics). Find the correlation between X and Y.

2. The joint density function for continuous random variables X and Y is given by

$$f(x, y) = \begin{cases} \frac{1}{3}(x + y) & 0 < x < 1, 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}.$$

   Find $\text{cov}(X, Y)$. Are X and Y independent?

3. Let X be a continuous random variable with probability density given by

$$f(x) = \begin{cases} 1 + x & -1 < x \leq 0 \\ 1 - x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}.$$

   Let $U = X$ and $V = X^2$. Show that $\text{cov}(U, V) = 0$.

4. Suppose 2 balls are removed (without replacement) from an urn containing n red balls and m blue balls, with $n, m \geq 2$. For $i = 1, 2$, let $X_i = 1$ if the ith ball removed is red and $X_i = 0$ if it is blue (i.e. not red).

   (a) Do you think $\text{cov}(X_1, X_2)$ is positive, negative or zero?

   (b) Compute $\text{cov}(X_1, X_2)$ to justify your answer to (a).

   (c) Suppose the red balls are numbered 1 through n. Let $Y_i = 1$ if red ball number i is removed, and $Y_i = 0$ otherwise. Do you think $\text{cov}(Y_1, Y_2)$ is positive, negative or zero?

   (d) Compute $\text{cov}(Y_1, Y_2)$ to justify your answer to (c).