MATH1550, Winter 2023
Mini-Assignment 8 – Expected Values

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1. Suppose X is a discrete random variable whose probability distribution function $f(x)$ is given by the following table.

x	-2	-1	0	1
$f(x)$	0.1	0.2	0.3	0.4

Find the expected value, $E(X)$, of X.

Round to 4 decimal places if needed.

Solution. We apply the definition of expected value for discrete random variables:

$$E(X) = \sum_x x f(x) = (-2) \cdot 0.1 + (-1) \cdot 0.2 + 0 \cdot 0.3 + 1 \cdot 0.4 = -0.2 - 0.2 + 0 + 0.4 = 0.$$

□

2. Suppose W is a continuous random variable with probability density function

$$g(w) = \begin{cases} \frac{3}{4}\left(1 - w^2\right) & w \in [-1, 1] \\ 0 & w \notin [-1, 1] \end{cases}.$$

Find the expected value, $E(W)$, of W.

Round to 4 decimal places if needed.

Solution. We apply the definition of expected value for continuous random variables:

$$\begin{aligned} E(W) &= \int_{-\infty}^{\infty} w g(w)\, dw = \int_{-\infty}^{-1} w \cdot 0\, dw + \int_{-1}^{1} w \cdot \frac{3}{4}\left(1 - w^2\right)\, dw + \int_{1}^{\infty} w \cdot 0\, dw \\ &= \int_{-\infty}^{-1} 0\, dw + \frac{3}{4} \int_{-1}^{1} \left(w - w^3\right)\, dw + \int_{1}^{\infty} 0\, dw = 0 + \frac{3}{4}\left(\frac{w^2}{2} - \frac{w^4}{4}\right)\Bigg|_{-1}^{1} + 0 \\ &= \frac{3}{4}\left(\frac{1^2}{2} - \frac{1^4}{4}\right) - \frac{3}{4}\left(\frac{(-1)^2}{2} - \frac{(-1)^4}{4}\right) = \frac{3}{4} \cdot \frac{1}{4} - \frac{3}{4} \cdot \frac{1}{4} = 0 \end{aligned}$$

□

3. A roulette wheel has 38 slots, numbered 0 through 36, with the remaining slot numbered 00. A ball is released into the spinning roulette wheel and ends up in a random slot, with equal likelihood for each slot, when the wheel spins down. Player wins \$1 if the slot in which the ball ends up is numbered 19-36 and wins nothing otherwise. What is the expected value of Player's winnings?

Round to 4 decimal places if needed.

Solution. There are 18 slots (19 through 36) out of 38 which result in Player winning \$1, and 20 slots out of 38 which result in Player winning \$0. If X is the random variable giving Player's winnings, it follows by the definition of expected value that:

$$\begin{aligned} E(X) &= \sum_x xf(x) = \sum_x xP(X = x) = 0 \cdot P(X = 0) + 1 \cdot P(X = 1) \\ &= 0 \cdot \frac{20}{38} + 1 \cdot \frac{18}{38} = \frac{18}{38} = \frac{9}{19} \approx 0.4737 \end{aligned}$$

□

4. Suppose the continuous random variable T has probability density function

$$f(t) = \begin{cases} 2t^{-3} & t \geq 1 \\ 0 & t < 1 \end{cases}.$$

Find the expected value, $E(T)$, of T,

Round to 4 decimal places if needed.

Solution. We apply the definition of expected value for continuous random variables.

$$\begin{aligned} E(T) &= \int_{-\infty}^{\infty} tf(t)\,dt = \int_{-\infty}^{1} t \cdot 0\,dt + \int_{1}^{\infty} t \cdot 2t^{-3}\,dt = \int_{-\infty}^{1} 0\,dt + 2\int_{1}^{\infty} t^{-2}\,dt \\ &= 0 + 2\lim_{a\to\infty}\int_1^a t^{-2}\,dt = 2\lim_{a\to\infty} \left.\frac{t^{-1}}{-1}\right|_1^a 2\lim_{a\to\infty} -t^{-1}\big|_1^a = 2\lim_{a\to\infty}\left[\left(-a^{-1}\right) - \left(-1^{-1}\right)\right] \\ &= 2\lim_{a\to\infty}\left[-\frac{1}{a} + 1\right] = 2\left[-0 + 1\right] = 2 \quad \text{since } \frac{1}{a} \to 0 \text{ as } a \to \infty \end{aligned}$$

□

5. The following game is available in a certain casino. The cards in a standard 52-card deck are assigned point values as follows: each numbered card (i.e. 2, 3, ..., 9, 10) is worth the number on the card, each face card (i.e. J, Q, K) is worth 10 points, and each ace is worth 1 point. A card is drawn at random from the deck and the player is paid the point value of the drawn card in dollars. What is the minimum that the casino should charge (to the nearest cent) for one play of the game to avoid losing money, at least on average?

Round to 2 decimal places if needed.

Solution. Note that there are 4 cards in the deck of each point value from 1 to 9 and 16 cards in the deck worth 10 points. If X is the point value of the randomly drawn card, it follows that it has its probability distribution function $f(x)$ given by $f(x) = P(X = x) = \frac{4}{52}$ for $x = 1, 2, \ldots, 9$ and $f(10) = P(X = 10) = \frac{16}{52}$. Applying the definition of expected value for discrete random variables, we get:

$$\begin{aligned} E(X) &= \sum_x xf(x) = 1 \cdot \frac{4}{52} + 2 \cdot \frac{4}{52} + \cdots + 9 \cdot \frac{4}{52} + 10 \cdot \frac{16}{52} \\ &= \frac{4}{52} \cdot (1 + 2 + \cdots + 9 + 10 \cdot 4) = \frac{4}{52} \cdot (45 + 40) = \frac{340}{52} = \frac{85}{13} \approx 6.5385 \end{aligned}$$

This means that a player can expect to win just under \$6.54 on average, so the casino ought to charge at least \$6.54 to avoid losing money, at least on average. ☐

6. Suppose X and Y are discrete random variables with their joint probability distribution given by the following table:

y \ x	-1	0	1
1	0.1		0.1
2	0.1	0.1	
3	0.2	0.1	
4	0.1	0.1	0.1

Find the expected value of $X - 2Y$.

Round to 4 decimal places if needed.

Solution. We apply the definition of expected value, adding up the value of $X - 2Y$ weighted by its probability for each cell in the table:

$$\begin{aligned}
E(X-2Y) &= \sum_y \sum_x (x-2y)P(X=x, Y=y) \\
&= (-1-2\cdot 1)\cdot 0.1 + (-2\cdot 1)\cdot 0 + (1-2\cdot 1)\cdot 0.1 \\
&\quad + (-1-2\cdot 2)\cdot 0.1 + (-2\cdot 2)\cdot 0.1 + (1-2\cdot 2)\cdot 0 \\
&\quad + (-1-2\cdot 3)\cdot 0.2 + (-2\cdot 3)\cdot 0.1 + (1-2\cdot 3)\cdot 0 \\
&\quad + (-1-2\cdot 4)\cdot 0.1 + (-2\cdot 4)\cdot 0.1 + (1-2\cdot 4)\cdot 0.1 \\
&= -0.3 + 0 - 0.1 - 0.5 - 0.4 + 0 - 1.4 - 0.6 + 0 - 0.9 - 0.8 - 0.7 = -5.7
\end{aligned}$$

☐