MATH1550, Winter 2023
Mini-Assignment 3 – Conditions

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1. A thick coin with a head on one side and a tail on the other has a probability of 0.4 of coming up heads, 0.4 of coming up tails, and 0.2 of coming to rest on its edge. What is the probability that exactly two tails will come up in three tosses of this coin? (Assume coin tosses are independent.)

   Round to 4 decimal paces if needed.

   Solution. To get exactly 2 tails, choose 2 of the 3 tosses to be tails (leaving the other toss to be either heads, or edge), thus there are $\binom{3}{2} = 3$ ways to get an outcome with exactly 2 tails. The probability of each of these is
   $$(0.4)(0.4)(0.6),$$
   therefore
   $$P(\text{exactly 2 tails}) = 3(0.4)(0.4)(0.6) = 0.288.$$
   $\square$

2. A fair coin is tossed four times. What is the probability of getting at least two heads among all four tosses, given that at least one head came up among the first two tosses?

   Round to 4 decimal paces if needed.

   Solution. The event that least one head came up among the first two tosses is
   $$\{HHHH, HHHT, HHTH, HTHH, THHH, HHTT,$$
   $$HTHT, HTTH, THHT, THTH, HTTT, THTT\}$$
   out of these 12 outcomes, all but 2 have at least 2 heads. Therefore the probability of getting at least two heads among all four tosses, given that at least one head came up among the first two tosses is
   $$\frac{10}{12} \approx 0.8333.$$
   $\square$

3. In a city of exactly $100,000$ people, 90% can expect to live to (at least) age 60 and 60% can expect to live to (at least) age 80. What is the probability that a resident of this city who is 60 years old will survive until they are 80?

   Round to 4 decimal paces if needed.

   Solution. Let A be the event that a person lives to at least 60, and B the event that a person lives to at least 80. Note that $B \subset A$. Then
   $$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)}{P(A)} = \frac{0.6}{0.9} = \frac{2}{3} \approx 0.6667$$
   $\square$

4. Recall that a five-card hand drawn from a standard 52-card deck is a *full house* if it consists of three of one kind (A, K, Q, J, 10, 9, 8, 7, 6, 5, 4, 3, 2) and a pair of another kind. A five-card hand is drawn all at once, so order doesn't matter, but is left face down. Two cards are then flipped over, revealing that one is $2\heartsuit$ and the other $2\spadesuit$. Knowing this, what is the probability that the hand is a full house?

Round to 4 decimal paces if needed.

Solution. Given that we already have a pair of 2's, we could make a full house by adding three of a kind, which has probability

$$\frac{\binom{12}{1}\binom{4}{3}}{\binom{50}{3}} = \frac{48}{19600}$$

or by adding another 2 (the $\diamondsuit$ or $\clubsuit$) and a pair of a different kind, which has probability

$$\frac{\binom{2}{1}\binom{12}{1}\binom{4}{2}}{\binom{50}{3}} = \frac{144}{19600}.$$

Adding these we get

$$\frac{12}{1225} \approx 0.0098$$

$\square$

5. One coin in a collection of 16 coins has two heads, the rest being fair coins with a head and a tail each. A coin is chosen at random from the collection and tossed four times, coming up with a head on each of the four tosses. What is the probability that the chosen coin is the two-headed one?

   Round to 4 decimal paces if needed.

   Solution. Let F be the event of choosing a fair coin, and thus F' is the event of choosing the two-headed coin. By the rule of total probability

$$\begin{aligned} P(\{HHHH\}) &= P(F)P(\{HHHH\}|F) + P(F')P(\{HHHH\}|F') \\ &= \left(\frac{15}{16}\right)\left(\frac{1}{16}\right) + \left(\frac{1}{16}\right)(1) \\ &= \frac{31}{256} \end{aligned}$$

   By Bayes' Theorem,

$$\begin{aligned} P(F'|\{HHHH\}) &= \frac{P(F' \cap \{HHHH\})}{P(\{HHHH\})} \\ &= \frac{P(F')P(\{HHHH\}|F')}{P(\{HHHH\})} \\ &= \frac{\left(\frac{1}{16}\right)(1)}{\frac{31}{256}} \\ &= \frac{16}{31} \\ &\approx 0.5161 \end{aligned}$$

$\square$

6. A fair coin is tossed four times. A is the event that at least one of the first three tosses was a head and B is the event that at least one of the last three tosses was a tail. Are A and B independent?

   Solution. We have

$$A' = \{TTTH, TTTT\}, \quad B' = \{HHHH, THHH\}, \quad A' \cap B' = \emptyset.$$

Now

$$P(A')P(B') = \left(\frac{2}{16}\right)\left(\frac{2}{16}\right) \neq 0 = P(A' \cap B)$$

and hence A' and B' are not independent. Using the fact that A and B are independent if and only if A' and B' are independent, we see that A and B are not independent. $\square$