Mathematics 1550H – Probability I: Introduction to Probability
TRENT UNIVERSITY, Winter 2023

Some Common Probability Distributions – *The Short Form**

Discrete Distributions

1. *Discrete Uniform.* n equally likely outcomes for some $n \geq 1$.
Probability function: $m(\omega) = \dfrac{1}{n}$.
Expected value and variance of a random variable X on Ω depend on just what values X assigns to each outcome $\omega \in \Omega$.

2. *Bernoulli Trial.* Two outcomes with probability of success p and of failure $q = 1 - p$. X counts successes.
Probability function: $m(1) = P(\text{success}) = p$ and $m(0) = P(\text{failure}) = q$.
Expected value: $\mu = E(X) = p$ *Variance:* $\sigma^2 = V(X) = pq$

3. *Binomial.* n Bernoulli trials, with probability of success p and of failure $q = 1 - p$. X counts successes.
Probability function: $m(k) = P(k \text{ successes}) = \dbinom{n}{k} p^k q^{n-k}$, where $0 \leq k \leq n$.
Expected value: $\mu = E(X) = np$ *Variance:* $\sigma^2 = V(X) = npq$

4. *Geometric.* Bernoulli trials repeated until the first success, with probability of success p and of failure $q = 1 - p$. X counts the number of trials required.
Probability function: $m(k) = P(\text{first success on } k\text{th trial}) = q^{k-1}p$, where $k \geq 1$.
Expected value: $\mu = E(X) = \dfrac{1}{p}$ *Variance:* $\sigma^2 = V(X) = \dfrac{q}{p^2}$

5. *Negative Binomial.* Bernoulli trials repeated until the kth success, with probability of success p and of failure $q = 1 - p$. X counts the number of trials required.
Probability function: $m(x) = P(k \text{ success on } x\text{th trial}) = \dbinom{x-1}{k-1} p^k q^{x-k}$
Expected value: $\mu = E(X) = \dfrac{k}{p}$ *Variance:* $\sigma^2 = V(X) = \dfrac{kq}{p^2}$

Continuous Distributions

1. *Continuous Uniform.*
Density function: $f(t) = \begin{cases} \frac{1}{b-a} & a \leq t \leq b \\ 0 & \text{otherwise} \end{cases}$
Expected value: $\mu = E(X) = \dfrac{a+b}{2}$ *Variance:* $\sigma^2 = V(X) = \dfrac{(b-a)^2}{12}$

2. *Exponential.*
Density function: $f(t) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}$
Expected value: $\mu = E(X) = \dfrac{1}{\lambda}$ *Variance:* $\sigma^2 = V(X) = \dfrac{1}{\lambda^2}$

* With apologies to the creators of *Buckaroo Banzai*. "Remember: No matter where you go, there you are."

3. *Standard normal.*

Density function: $\varphi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$

Expected value: $\mu = E(X) = 0$ *Variance:* $\sigma^2 = V(X) = 1$

4. *Normal.* … with mean μ and standard deviation σ.

Density function: $f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(t-\mu)^2/2\sigma^2}$

Expected value: $E(X) = \mu$ *Variance:* $V(X) = \sigma^2$