MATH1550, Winter 2023:
Exercise Set 8

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1. Let X be a discrete random variable with the following probability distribution:

$$P(X=0)=\frac{1}{3},\quad P(X=1)=P(X=6)=\frac{1}{165},$$

$$P(X=2)=P(X=5)=\frac{1}{11},\quad P(X=3)=P(X=4)=\frac{13}{55}$$

Find $E(X)$.

2. Two coins are tossed. The first coin has a probability of 0.6 that it will land on heads, and the second coin has a probability of 0.7 that it will land on heads. Let X be the total number of heads.

   (a) What is the range of X?

   (b) Find the probability distribution for X.

   (c) Compute $E(X)$.

3. You are playing a dice game where two (regular) dice are rolled and you are paid the amount shown (the sum of the two dice) in dollars. If the game costs $7 to play, what can you expect to win or lose; i.e. what is the expected value of this game?

4. You run a business buying and selling coconuts. You have $1,000, and coconuts are currently selling for $2 each. In one week you can sell the coconuts, but the price will change to either half the price ($1) or double the price ($4), with each of these being equally likely.

   (a) If your goal is maximize the *expected* amount of money you have after a week (i.e. after you are able to sell) how many coconuts should you buy at $2 each?

   (b) If your goal is maximize your *expected* number of coconuts after a week, how many coconuts should you buy at $2 each (vs. buying a week later at the new price)?

5. A game of chance is called *fair*, if each player's expected value is zero. If the casino pays us $10 for rolling a 3 or a 4 with a regular 6-sided die, what should we have to pay for rolling a 1,2,5, or 6, in order to make this a fair game?

6. The probability density of X is given by

$$f(x)=\begin{cases}\frac{1}{8}(x+1) & \text{for } 2\le x\le 4\\ 0 & \text{otherwise.}\end{cases}$$

Find the mean μ of X. (Note that $\mu=E(X)$.)

7. Let X be a random variable with the following distribution

x	-2	-1	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Let $Y=X^2$.

(a) Find the distribution $g(y)$ of Y.

(b) Find the joint distribution $f(x, y)$ of X and Y.

(c) Find the expected value of $2X + Y$.

(d) Find $E(X)$, $E(Y)$ and $E(XY)$. Note that this example shows that $E(XY) = E(X)E(Y)$, however X and Y are not independent.

8. The joint probability density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{2}{7}(x + 2y) & \text{for } 0 \le x \le 1, 1 < y < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

Find the expected value of $g(X, Y) = \frac{X}{Y^3}$.

9. The probability density of X is given by

$$f(x) = \begin{cases} \frac{x}{2} & \text{for } 0 < x \le 1 \\ \frac{1}{2} & \text{for } 1 < x \le 2 \\ \frac{3-x}{2} & \text{for } 2 < x < 3 \\ 0 & \text{elsewhere.} \end{cases}$$

Find the expected value of $g(X) = X^2 - 5X + 3$.

10. The probability that Ms. Brown will sell a piece of property at a profit of \$3,000 is $\frac{3}{20}$, the probability that she will sell at a profit of \$1,500 is $\frac{7}{20}$, the probability that she will break even is $\frac{7}{20}$ and the probability that she will lose \$1,500 is $\frac{3}{20}$. What is her expected profit?

11. The joint probability density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y) & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find $E(XY)$.

12. The number of minutes that a flight from Phoenix to Tucson is early or late is a continuous random variable with probability density

$$f(x) = \begin{cases} \frac{1}{243}(36 - x^2) & \text{for } -6 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

If the posted arrival time is 12:00pm, find the expected arrival time. (Take negative values to mean early, positive values to mean late)

13. Find the expected value for a random variable X with probability density function given by

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$$

14. Let X be the number of points rolled with a regular 6-sided die. Find the expected value of $3X^2+2X-1$.

15. Let X be a discrete random variable with the probability distribution given below. Find the expected value of X.

x	$f(x)$
-2	$\frac{1}{20}$
-1	$\frac{3}{20}$
0	$\frac{6}{20}$
1	$\frac{2}{20}$
2	$\frac{7}{20}$
3	$\frac{1}{20}$

16. Let X be a discrete random variable with the probability distribution given below. Find $E(X^2)$.

x	$f(x)$
-2	$\frac{1}{20}$
-1	$\frac{3}{20}$
0	$\frac{6}{20}$
1	$\frac{2}{20}$
2	$\frac{7}{20}$
3	$\frac{1}{20}$

17. Let X be a continuous random variable with the probability density given below. Find $E(X)$.

$$f(x) = \begin{cases} \frac{x}{2} & \text{for } 0 \leq x \leq 1 \\ \frac{1}{2} & \text{for } 1 < x \leq 2 \\ \frac{3-x}{2} & \text{for } 2 < x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

18. A game of chance is called *fair* if each player's expected profit is zero. Consider a casino game where the player rolls two fair dice and wins the sum shown on the 2 dice (in dollars). How much should the casino charge the player in order to make this a fair game?

19. A game of chance is called *fair* if each player's expected profit is zero. Suppose you are making wagers with your friend and you tell them that they have to pay you \$10 if they roll a 3 or a 4 with fair 6-sided die. In order to make the game fair, how much should your promise to pay your friend if they roll a 1, 2, 5, or 6?

20. The joint distribution for X and Y is given below. Find $E(X+Y)$.

		x	
		0	1
y	0		$\frac{1}{8}$
	1	$\frac{1}{8}$	$\frac{2}{8}$
	2	$\frac{2}{8}$	$\frac{1}{8}$
	3	$\frac{1}{8}$	