MATH1550, Winter 2023:
Exercise Set 7

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1. Let X and Y be discrete random variables with joint probability distribution given by the following table:

		x			
		1	2	3	4
y	0	$\frac{4}{84}$	$\frac{18}{84}$	$\frac{12}{84}$	$\frac{1}{84}$
	1	$\frac{12}{84}$	$\frac{24}{84}$	$\frac{6}{84}$	
	2	$\frac{4}{84}$	$\frac{3}{84}$		

   (a) Find the marginal distributions for X and Y.

   (b) Find the conditional distribution for X given $Y = 1$.

2. A fair coin is tossed twice. Let X and Y be random variables such that

   - $X = 1$ if the first toss is heads, and $X = 0$ otherwise.
   - $Y = 1$ if both tosses are heads, and $Y = 0$ otherwise

   (a) Give the joint probability distribution for X and Y

   (b) Find the marginal distributions for X and Y.

   (c) Determine whether or not X and Y are independent.

3. Let X and Y be discrete random variables with joint probability distribution given by the following table:

		x		
		2	3	4
y	1	0.06	0.15	0.09
	2	0.14	0.35	0.21

   (a) Find the marginal distributions for X and Y.

   (b) Find the conditional distribution for X given $Y = 2$.

   (c) Determine whether or not X and Y are independent.

4. Let X be a random variable with the following distribution

x	-2	-1	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Let $Y = X^2$.

(a) Find the distribution $g(y)$ of Y.
(b) Find the joint distribution $f(x, y)$ of X and Y.
(c) Find the marginal distributions of X and Y.
(d) Determine whether or not X and Y are independent.

5. The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} x+y & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the marginal densities for X and Y, and determine whether X and Y are independent.

6. Find the marginal densities of X and Y given their joint probability density

$$f(x,y) = \begin{cases} \frac{2}{5}(x+4y) & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

7. Let X and Y be jointly continuous random variables with joint probability density given by

$$f(x,y) = \begin{cases} \frac{12}{5}(2x - x^2 - xy) & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal densities for X and Y.
(b) Find the conditional density for X given $Y = y$ and the conditional density for Y given $X = x$.
(c) Compute the probability $P(\frac{1}{2} < X < 1 | Y = \frac{1}{4})$.
(d) Determine whether or not X and Y are independent.

8. Let X and Y be discrete random variables with joint probability distribution given by the following table:

		x		
		-3	2	4
y	1	0.1	0.2	0.2
	3	0.3	0.1	0.1

(a) Find the conditional distribution for X given $Y = 1$.
(b) Are X and Y independent? Justify your answer.

9. Given the joint probability density

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the conditional distribution of X given $Y = y$ and use it to evaluate $P(X \leq \frac{1}{2} | Y = \frac{1}{2})$.

10. The joint probability density function for continuous random variables is given below. Let $f(x|y)$ be the conditional density for X given $Y = y$. Find $P(0 \leq X \leq \frac{1}{2}|Y = 1)$.

$$f(x,y) = \begin{cases} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) & \text{for } 0 < x < 1, 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$