1. Let X and Y be discrete random variables.

Determine whether this table corresponds to a valid joint probability distribution.

2. Let X and Y be discrete random variables with joint probability distribution given by the following table:

- (a) Determine the appropriate value for $k \in \mathbb{R}$ so that is a valid joint probability distribution.
- (b) Find the following probabilities
 - P(X = 2, Y = 3)
 - $P(X \le 2, Y = 1)$
 - P(X < 2, Y = 1)
 - $P(X > 3, Y \le 3)$
 - P(X=2)
 - $P(Y \leq 3)$
- 3. A fair coin is tossed twice. Let X and Y be random variables such that
 - X = 1 if the first toss is heads, and X = 0 otherwise.
 - Y = 1 if both tosses are heads, and Y = 0 otherwise

Give the joint probability distribution for X and Y.

4. The joint probability density of continuous random variables X and Y is given by

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & \text{for } 0 < x < 1, 0 < y < 1\\ 0 & \text{elsewhere} \end{cases}$$

- (a) Verify that this is a valid joint probability density function.
- (b) Find the following probabilities
 - $P(0 \le X \le 1, 0.5 \le Y < 1)$
 - $P(0.25 \le X \le 0.5, 0 \le Y < 1)$

5. Let X and Y be continuous random variables defined on a joint sample space. Consider the function

$$f(x,y) = \begin{cases} 2(x+4y) & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Show that is is not a valid joint probability density of continuous random variables X and Y. Find an appropriate constant scaling factor to "salvage" this function.

6. The joint probability density of continuous random variables X and Y is given by

$$f(x,y) = \begin{cases} x+y & \text{for } 0 < x < 1, 0 < y < 1\\ 0 & \text{elsewhere} \end{cases}$$

Determine the joint cumulative distribution function, and find $P(X < \frac{1}{2}, Y < 1)$.

7. The joint probability density of continuous random variables X and Y is given by

$$f(x) = \begin{cases} \frac{2}{55}(x+27) & \text{for } 0 \le x \le 1, 1 < y < 2\\ 0 & \text{elsewhere} \end{cases}$$

- (a) Verify that this is a valid joint probability density function.
- (b) Find the following probabilities
 - $P(0 \le X \le 1, 1.5 \le Y < 2)$
 - $P(0.25 \le X \le 0.5, 1 \le Y < 2)$
 - $P(0.5 \le X \le 1, 1.25 \le Y < 1.5)$
- (c) Find the joint cumulative distribution function.
- 8. The joint probability density of continuous random variables X and Y is given by

$$f(x,y) = \begin{cases} \frac{2}{5}(2x+3y) & \text{for } 0 < x < 1, 0 < y < 1\\ 0 & \text{elsewhere} \end{cases}$$

- (a) Verify that this is a valid joint probability density function.
- (b) Find the joint cumulative distribution function.
- (c) Use part (b) to find
 - $P(X \le 1, Y \le 0.5)$
 - $P(0.25 < X \le 0.5, Y \le 1)$
 - $P(0.25 \le X \le 0.5, 0.5 < Y \le 1)$
- 9. Let X and Y be discrete random variables with joint probability distribution given by the following table:

		-3	$\frac{x}{2}$	4
y	1	0.1	0.2	0.2
	3	0.3	0.1	0.1

Find the marginal distributions for X and Y.

10. The joint distribution function, f(x, y), for discrete random variables X and Y is given below. Find F(3,3) where F(x, y) is the cumulative distribution function for X and Y.

			x
		1	2
y	-2	0.1	0.2
	-1	0.2	0.1
	4	0	0.1
	5	0.3	0

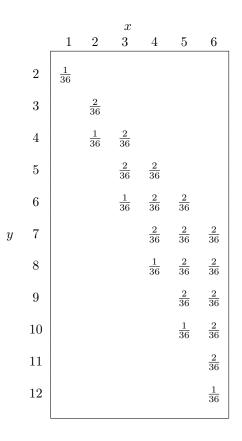
- 11. A fair coin is tossed 4 times. Let random variable X be the number of heads appearing in the 4 tosses and Y be the largest number of consecutive heads in the 4 tosses. If f(x, y) is the joint probability distribution for X and Y, find f(3, 2). (For practice find the entire joint distribution.)
- 12. The joint probability density function for continuous random variables is given below. Find $P(0 \le X \le \frac{1}{2}, \frac{1}{2} \le Y \le 1)$.

$$f(x,y) = \begin{cases} 12xy(1-x) & \text{ for } 0 < x < 1, 0 < y < 1\\ 0 & \text{ elsewhere} \end{cases}$$

13. Is the following function a valid joint density function?

$$f(x,y) = \begin{cases} \frac{x+y}{2} & \text{for } 0 < x < 1, 0 < y < 1\\ 0 & \text{elsewhere} \end{cases}$$

14. The joint distribution, f(x, y), for discrete random variables X and Y is given below. Let g(x) be the marginal distribution for X. Find g(4).



15. The joint probability density function for continuous random variables is given below. Find g(x), the marginal density for X.

$$f(x,y) = \begin{cases} \frac{6}{7} \left(x^2 + \frac{xy}{2}\right) & \text{for } 0 < x < 1, 0 < y < 2\\ 0 & \text{elsewhere} \end{cases}$$