MATH1550, Winter 2023:
Exercise Set 4

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1. Suppose a coin is weighted so that the probability of getting heads on any flip is twice the probability of getting tails. The coin is tossed 3 times. Let X be the random variable which assigns total number of heads to an outcome.

   (a) Give the range of X and find $P(X = x)$ for each x in the range of X.

   (b) Find the cumulative distribution for X.

   (c) Draw a probability histogram for X.

2. Which of the following functions can be used as a valid probability distribution function?

$$A: \quad f(x) = \frac{x-2}{5} \quad \text{for } x = 1, 2, 3, 4, 5$$

$$B: \quad f(x) = \frac{x^2}{30} \quad \text{for } x = 1, 2, 3, 4$$

$$C: \quad f(x) = \frac{x^2}{30} \quad \text{for } x = 0, 1, 2, 3, 4$$

$$D: \quad f(x) = \frac{1}{5} \quad \text{for } x = 0, 1, 2, 3, 4, 5$$

$$E: \quad f(x) = \frac{x}{15} \quad \text{for } x = 1, 2, 3, 4, 5$$

$$F: \quad f(x) = \frac{\binom{5}{x}}{32} \quad \text{for } x = 0, 1, 2, 3, 4, 5$$

3. Determine an appropriate value for k so that

$$f(x) = \frac{k}{x} \quad \text{for } x = 1, 2, 3, 4, 5$$

   is a valid probability distribution. *(Assume $f(x) = 0$ for all other values of x.)*

4. A fair 4-sided die (with sides numbered $1, 2, 3, 4$) and a fair 8-sided die (with sides numbered $1, 2, 3, 4, 5, 6, 7, 8$) are rolled. Outcomes of the individual dice are independent. Let Y be the random variable that gives the sum of the two dice. Give the range and probability distribution of Y.

5. Three (regular) dice are thrown and the $6^3 = 216$ possible outcomes are equally likely. Let X be the random variable whose value is the sum of the three dice. What is the range of X?

6. The cumulative distribution for discrete random variable X is

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \\ \frac{1}{3} & \text{for } x \in [1, 4) \\ \frac{1}{2} & \text{for } x \in [4, 6) \\ \frac{5}{6} & \text{for } x \in [6, 10) \\ 1 & \text{for } x \geq 10 \end{cases}$$

   (a) Find $P(X = 4)$.
   (b) Find $P(2 < X \leq 6)$.

7. Suppose the cumulative distribution for a random variable X is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \leq x < 1 \\ \frac{5}{8} & 1 \leq x < 2 \\ \frac{11}{12} & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

   (a) Give the probability distribution for X.
   (b) Use $F(x)$ to find $P(\frac{1}{2} < X < \frac{5}{2})$.
   (c) Draw a probability histogram for X.

8. A fair 4-sided die (with sides numbered $1, 2, 3, 4$) and a fair 6-sided die (with sides numbered $1, 2, 3, 4, 5, 6$) are rolled. Outcomes of the individual dice are independent. Let Y be the random variable that gives the sum of the two dice.

   (a) What is range of Y?
   (b) Give the probability distribution for Y (you don't need a formula).
   (c) Give the cumulative distribution function for Y.

9. Two balls are chosen randomly without replacement from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win \$2 for each black ball selected and we lose \$1 for each white ball selected. Let X denote our winnings.

   (a) What is the range of X?
   (b) Find the probability distribution of X.
   (c) Find the cumulative distribution of X.

10. Suppose discrete random variable X has range $\{0, 1, 2\}$ with probability distribution

$$f(x) = \frac{\binom{2}{x}\binom{4}{3-x}}{\binom{6}{3}}.$$

   (a) Verify that this is a valid probability distribution.
   (b) Create a histogram for this probability distribution.
   (c) Give the cumulative probability distribution for X.
   (d) Come up with an example of a probability experiment which corresponds to this X.

11. Suppose you have 5 cards which are numbered 1 to 5. You draw 2 of them at random without replacement. Let random variable X be the smallest number out the two cards you have drawn. Find $P(X = 2)$.

12. In a certain dice rolling game, the player rolls two fair six-sided dice and wins \$3 if the sum of the dice is a multiple of 3, \$5 if the sum of dice is a multiple of 5 and \$7 if the sum of the dice is a multiple of 7. Let random variable Y denote the amount of money won on a single roll of both dice. Then Y has range $\{0, 3, 5, 7\}$. Find the probability distribution for Y.

    Fill in the blanks:

$$P(Y = 0) = \_\_ \quad P(Y = 3) = \_\_ \quad P(Y = 5) = \_\_ \quad P(Y = 7) = \_\_$$