MATH1550, Winter 2023:
Exercise Set 2

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1. Rolling a pair of dice with equally likely outcomes, what is the probability of rolling a 5, a 7 or a 12? (*Here we want the sum of the two dice.*)

2. Rolling a pair of dice with equally likely outcomes, what is the probability of rolling a 1, a 4 or an 11? (*Here we want the sum of the two dice.*)

3. Rolling a pair of dice with equally likely outcomes, what is the probability of rolling at least 8? (*Here we want the sum of the two dice.*)

4. An experiment has 5 possible outcomes S_1, S_2, S_3, S_4 and S_5 (which are mutually exclusive). Which of the following are valid ways to assign probabilities?

$$A: \quad P(S_1) = 0.20, \quad P(S_2) = 0.20, \quad P(S_3) = 0.20, \quad P(S_4) = 0.20, \quad P(S_5) = 0.20$$

$$B: \quad P(S_1) = 0.21, \quad P(S_2) = 0.26, \quad P(S_3) = 0.58, \quad P(S_4) = 0.01, \quad P(S_5) = 0.06$$

$$C: \quad P(S_1) = 0.18, \quad P(S_2) = 0.19, \quad P(S_3) = 0.20, \quad P(S_4) = 0.21, \quad P(S_5) = 0.22$$

$$D: \quad P(S_1) = 0.10, \quad P(S_2) = 0.30, \quad P(S_3) = 0.10, \quad P(S_4) = 0.60, \quad P(S_5) = -0.10$$

$$E: \quad P(S_1) = 0.23, \quad P(S_2) = 0.12, \quad P(S_3) = 0.05, \quad P(S_4) = 0.5, \quad P(S_5) = 0.08$$

5. An experiment has four possible outcomes A, B, C, D that are mutually exclusive. Are either of the following assignments of probabilities permissible?

   (a) $P(A) = 0.72, P(B) = 0.13, P(C) = 0.25, P(D) = -0.10$
   (b) $P(A) = \frac{3}{111}, P(B) = \frac{65}{111}, P(C) = \frac{22}{111}, P(D) = \frac{21}{111}$

6. For events A and B we have that $P(A) = 0.45$, $P(B) = 0.33$ and $P(A \cap B) = 0.12$. Find $P(A \cup B)$.

7. Suppose a random experiment has sample space $S = \{a, b, c, d\}$ with

$$P(a) = 0.2, \quad P(b) = 0.3, \quad P(c) = 0.4, \quad P(d) = 0.1.$$

   Consider the events $A = \{a, b\}$ and $B = \{b, c, d\}$ and determine the following probabilities:

   (a) $P(A)$
   (b) $P(B)$
   (c) $P(A')$
   (d) $P(A \cup B)$
   (e) $P(A \cap B)$

8. For events A and B we have that $P(A) = 0.72$, $P(B) = 0.19$ and $P(A \cap B) = 0.08$. Find $P(A' \cap B')$.

9. Given $P(A) = 0.59$, $P(B) = 0.3$ and $P(A \cap B) = 0.21$ find $P(A \cap B')$.

10. A 52-card deck is divided into 13 *ranks*, $A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K$, each having 4 *suits*, $\clubsuit, \diamondsuit, \heartsuit, \spadesuit$. What is the probability of drawing a six-card hand from a standard deck of 52 cards with: four-of-a-kind, and one pair?

11. What is the probability of drawing a five-card poker hand of: two (different) pairs?

12. What is the probability of drawing a seven-card hand from a deck of 52 cards with: three-of-a-kind, and two (different) pairs?

13. Two cards are randomly drawn from a standard deck of 52 cards. What is the probability the rank of both cards will be greater than 3 but less than 8?

14. For events A, B and C we have that $P(A) = 0.62$, $P(B) = 0.31$, $P(C) = 0.50$, $P(A \cap B) = 0.09$, $P(A \cap C) = 0.34$, $P(B \cap C) = 0.16$ and $P(A \cap B \cap C) = 0.01$. Find $P(A \cup B \cup C)$.

15. Six regular dice are thrown at the same time. *(regular means six-sided with each outcome equally likely)*

    (a) Describe the sample space of this experiment and count how many possible outcomes there are. *(you might wish to think of the dice as being six different colours)*

    (b) What is the probability of rolling a *straight*, i.e. ⚀⚁⚂⚃⚄⚅? *(in any order)*

    (c) What is the probability of rolling a *full house*; three-of-a-kind with two-of-(another)kind, plus a sixth number different form those two kinds? For example ⚀⚀⚀⚁⚁⚂. *(in any order)*

16. Three coins, with sides marked H and T, are tossed.

    (a) List all possible outcomes. What is the probability of any outcome given that they are all equally likely?

    (b) If we are only interested in the total number of heads that appear in this experiment, then we can take the sample space to be $S = \{0, 1, 2, 3\}$. Given that the outcomes of part (a) are equally likely, what are the probabilities for each element of S?

17. Alice, Bob, Celia and Dan apply for job opening, where one of the four will be selected. Alice is twice as likely to be selected as Bob. Bob and Celia have an equally likely change of being selected, while Celia is twice as likely to be selected as Dan. What is the probability that Alice will not be selected?

18. Three horses, A, B, and C, are in a race. Horse A is twice as likely to win as B, and B is twice as likely to win as C.

    (a) Find the probabilities $P(A)$, $P(B)$, and $P(C)$ of each horse winning.

    (b) What is the probability that B or C wins?

19. A regular coin and 6-sided die are tossed. We will express the sample space for this experiment with the set

$$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

   (a) Express each of the following events as subsets of S:
      - $A = \{\text{heads with an even number}\}$
      - $B = \{\text{the number is less than 3}\}$
      - $C = \{\text{tails and an odd number}\}$

   (b) If all outcomes in S are equally likely, find $P(A)$, $P(B)$ and $P(C)$.

   (c) Express the following events as subsets of S:
      - A or B occurs
      - B and C occur.
      - Only B occurs.

   (d) Which pairs of events A, B and C are mutually exclusive?

20. Let A and B be events from a sample space S. Find an expression (using unions, intersections and complements) for the following events, and draw a Venn diagram representing the event.
   - A but not B
   - Neither A nor B.
   - Either A or B but not both.

21. Let A, B and C be events from a sample space S. Find an expression (using unions, intersections and complements) for the following events, and draw a Venn diagram representing the event.
   - A and B but not C occur
   - Only A occurs

22. A six-sided die, with sides numbered 1 through 6, is weighted so that the probability of rolling a given number is as follows:

x	$P(x)$
1	0.1
2	0.3
3	0.2
4	0.1
5	0.1
6	0.2

   (a) Verify that P is a valid probability measure for the experiment (assuming countable additivity).

   (b) Consider the following events:

$$A = \{\text{even number}\}, \quad B = \{2, 3, 4, 5\}, \quad C = \{x : x < 3\}, \quad D = \{x | x > 7\}.$$

   Find:
      - $P(A)$
      - $P(B)$
      - $P(C)$
      - $P(D)$

23. Five valentines are randomly placed into five letter boxes. What is the probability that exactly two of the five boxes will be empty?

24. There is a 0.08 probability that Andy is absent from work, and a 0.05 probability that Blake is absent from form work. The probability that both will be absent is 0.02. What is the probability that only one of the two will be absent at work?