

MATH1550, Winter 2023:
Exercise Set 11

1. If X is a Bernoulli random variable, where θ is the probability of a success, show that

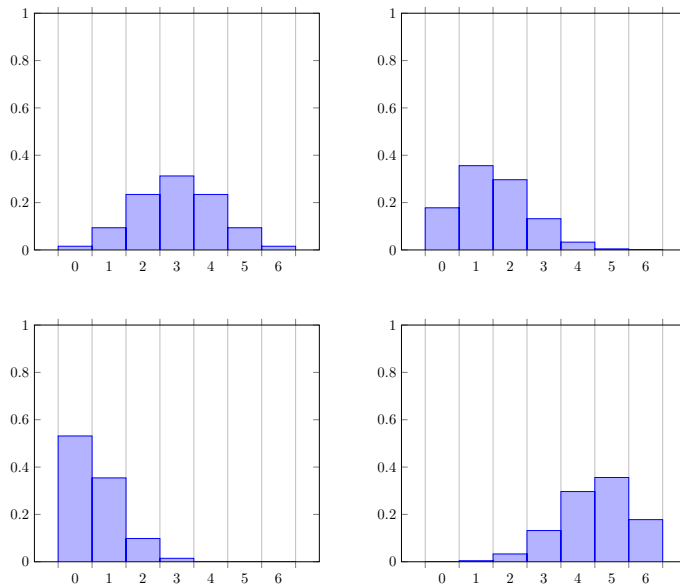
- (a) $\mu = \theta$
- (b) $\sigma^2 = \theta(1 - \theta)$.

2. If X is a Bernoulli random variable, where θ is the probability of a success. Find the cumulative distribution function $F(x)$ for X .

3. Let X be a random variable with binomial distribution $b(x; n, \theta)$. Prove that

$$b(x; n, \theta) = b(n - x; n, 1 - \theta).$$

4. For each graph of $b(x; n, \theta)$ we have $n = 6$. Determine which of these has $\theta = 0.1, 0.25, 0.5$, and 0.75



5. Let X have binomial distribution $b(x; 12, 0.25)$. Find

- (a) $P(X = 6)$
- (b) $P(3 \leq X \leq 5)$

6. Let X have binomial distribution $b(x; 30, 0.3)$ find $P(X = 10)$.

7. Draw the probability histogram for a random variable X with binomial distribution $b(x; 7, 0.35)$. (*You will need to use approximate values here.*)

8. You are fishing on your favourite lake. For a single cast of your fishing rod you know there is a 45% chance of catching a walleye, a 25% chance of catching a pike, and a 30% chance of catching a bass. You are patient and always catch something on a cast.
- What is the probability of catching exactly 12 bass on 20 casts?
 - How many walleye do you expect to catch in 20 casts?
 - What is the probability that you catch exactly 8 pike in 15 casts?
9. Suppose that it is known that on any given day in the month of March there is a 0.3 probability of rain. Find the mean number of rainy days in March and the standard deviation.
10. Your friend claims that if she rolls a regular 6-sided die 6 times, it is more likely than not that at least one time the outcome will be a 6. Is she right?
11. Suppose X has discrete uniform distribution, where the range of X is $\{1, 2, \dots, 24\}$. Find the mean of X .
12. Suppose X has discrete uniform distribution, where the range of X is $\{0, 2, 4, 6, \dots, 2k\}$ for some positive integer k . Find the mean of X .
13. A multiple choice test has 8 questions where there are 3 options, A , B or C , per question (and only 1 is correct). If a student decides to write the test by choosing A , B or C at random for each question, what is the probability that they get exactly 4 questions correct?
14. A dart player can hit the bullseye with a probability of 0.25 on a given shot. What is the probability that they will hit the bullseye more than 4 times in 6 shots.
15. A dart player can hit the bullseye with a probability of 0.25 on a given shot. What is the minimum number of darts that must be thrown, so that there is at least a 50% probability that the bullseye gets hit. *Hint: Consider the probability that the bullseye does not get hit.*
16. Let X be a random variable with binomial distribution $b(x; 15, 0.2)$. Find the probability that X lies within 2 standard deviations of its mean; i.e. compute $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$. To do this, make use of the table below.

n	k	$\binom{n}{k}$	0.01	0.05	0.10	0.15	0.20	0.25	0.30	1/3	0.35	0.40	0.45	0.49	0.50
15	0	1	0.8601	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0023	0.0016	0.0005	0.0001	0.0000	
	1	15	0.1303	0.3658	0.3432	0.2312	0.1319	0.0668	0.0305	0.0171	0.0126	0.0047	0.0016	0.0006	0.0005
	2	105	0.0092	0.1348	0.2669	0.2856	0.2309	0.1559	0.0916	0.0599	0.0476	0.0219	0.0090	0.0040	0.0032
	3	455	0.0004	0.0307	0.1285	0.2184	0.2501	0.2252	0.1700	0.1299	0.1110	0.0634	0.0318	0.0166	0.0139
	4	1365		0.0049	0.0428	0.1156	0.1876	0.2252	0.2186	0.1948	0.1792	0.1268	0.0780	0.0478	0.0417
	5	3003		0.0006	0.0105	0.0449	0.1032	0.1651	0.2061	0.2143	0.2123	0.1859	0.1404	0.1010	0.0916
	6	5005		0.0000	0.0019	0.0132	0.0430	0.0917	0.1472	0.1786	0.1906	0.2066	0.1914	0.1617	0.1527
	7	6435			0.0003	0.0030	0.0138	0.0393	0.0811	0.1148	0.1319	0.1771	0.2013	0.1997	0.1964
	8	6435				0.0005	0.0035	0.0131	0.0348	0.0574	0.0710	0.1181	0.1647	0.1919	0.1964
	9	5005				0.0001	0.0007	0.0034	0.0116	0.0223	0.0298	0.0612	0.1048	0.1434	0.1527
	10	3003					0.0001	0.0007	0.0030	0.0067	0.0096	0.0245	0.0515	0.0827	0.0916
	11	1365						0.0001	0.0006	0.0015	0.0024	0.0074	0.0191	0.0361	0.0417
	12	455							0.0001	0.0003	0.0004	0.0016	0.0052	0.0116	0.0139
	13	105									0.0001	0.0003	0.0010	0.0026	0.0032
	14	15											0.0001	0.0004	0.0005
15	1														