MATH1550, Winter 2023:
Exercise Set 10

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1. Use properties of expected value to prove that $\text{cov}(X, Y)$ (or σ_{XY}) is given by

$$\text{cov}(X, Y) = E(XY) - E(X)E(Y).$$

2. Show, for the case of joint discrete random variables X and Y, that if X and Y are independent then

$$E(XY) = E(X)E(X).$$

(*find an example in the notes/exercises where the converse is not true.*)

3. Let X and Y be discrete random variables with joint probability distribution given by the following table:

		x		
		-1	0	1
y	0	0	1/6	1/12
	1	1/4	0	1/2

(a) Find the covariance of X and Y.

(b) Determine whether X and Y are independent (justify your answer).

4. Let X and Y be jointly continuous random variables with joint probability density given by

$$f(x, y) = \begin{cases} \frac{3}{5}x(y + x) & \text{for } 0 < x < 1, 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find μ_X and μ_Y.

(b) Find the covariance of X and Y. Are X and Y independent?

5. Let X and Y be continuous random variables with joint probability density

$$f(x, y) = \begin{cases} \frac{1}{8}(x + y) & \text{for } 0 \leq x \leq 2, 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal distribution for X.

(b) Find the covariance for X and Y.

6. Let X and Y be continuous random variables with joint probability density

$$f(x, y) = \begin{cases} 2x & \text{for } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal distribution for Y.

(b) Find the covariance for X and Y.

7. The joint distribution, $f(x, y)$, for discrete random variables X and Y is given below. Find the covariance of X and Y.

y \ x	1	2	3	4	5	6
2	$\frac{1}{36}$					
3		$\frac{2}{36}$				
4		$\frac{1}{36}$	$\frac{2}{36}$			
5			$\frac{2}{36}$	$\frac{2}{36}$		
6			$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	
7				$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$
8				$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$
9					$\frac{2}{36}$	$\frac{2}{36}$
10					$\frac{1}{36}$	$\frac{2}{36}$
11						$\frac{2}{36}$
12						$\frac{1}{36}$

8. Let X and Y have joint density function given below. Find $E(X)$.

$$f(x, y) = \begin{cases} \frac{x+y}{3} & \text{for } 0 < x < 2, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

9. Let X and Y have joint density function given below. Given that $E(X) = \frac{5}{6}$ and $E(Y) = \frac{17}{6}$, find $\text{Cov}(X, Y)$.

$$f(x, y) = \begin{cases} \frac{6-x-y}{8} & \text{for } 0 < x < 2, 2 < y < 4 \\ 0 & \text{elsewhere} \end{cases}$$

10. Let X and Y be joint continuous random variables with joint density

$$f(x, y) = \begin{cases} \frac{2}{3}(x + 2y) & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the conditional expected value of X given $Y = \frac{1}{2}$, i.e. find $E(X|\frac{1}{2})$.

11. Let X be the amount a salesperson spends on gas in a day, and Y be the amount of money for which they are reimbursed. The joint density of X and Y is

$$f(x,y) = \begin{cases} \frac{1}{25}\left(\frac{20-x}{x}\right) & \text{for } 10 < x < 20, \frac{x}{2} < y < x \\ 0 & \text{otherwise} \end{cases}$$

(gives the probability (density) that they will be reimbursed y dollars after spending x dollars)

Find, $f(y|x)$, the conditional probability of Y given $X = x$ and use it to find the probability of being reimbursed at least \$8 given that \$12 was spent. What is the expected reimbursement given that \$12 was spent?

12. Let X and Y have joint density function given below. Find $E(Y|X = 1)$. Hint, the marginal density for X is $g(x) = \frac{3-x}{4}$ for $0 < x < 2$ and is 0 elsewhere.

$$f(x,y) = \begin{cases} \frac{6-x-y}{8} & \text{for } 0 < x < 2, 2 < y < 4 \\ 0 & \text{elsewhere} \end{cases}$$