Trent University, Winter 2018

MATH 1550H Test #2

Friday, 16 March

Time: 50 minutes

Instructions

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator, a standard normal table, and an aid sheet.

1. Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

a. A fair standard die is rolled until either 1 or 2 comes up. What are the expected value $E(Y)$ and variance $V(Y)$ of the random variable Y that counts the number of rolls that occur in this experiment?

b. Verify that $f(x) = \begin{cases} \dfrac{2x}{(1+x^2)^2} & x \geq 0 \\ 0 & x \leq 0 \end{cases}$ is a valid density function.

c. A fair coin is tossed three times. The random variable U counts how many tails came up in the three tosses and the random variable V counts how many heads came up on the second of the three tosses. Determine whether U and V are independent or not.

2. Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

a. Find the expected value $E(W)$ and variance $V(W)$ of the continuous random variable W that has as its density function $h(w) = \begin{cases} \frac{3}{4}\left(1-w^2\right) & -1 \leq w \leq 1 \\ 0 & \text{otherwise} \end{cases}$.

b. A fair four-sided die with faces numbered 0, 2, 3, and 5, respectively, is rolled once. What are the expected value and variance of the number that comes up?

c. Suppose that the continuous random variable Z has a standard normal distribution. Find $P(Z > 1.26)$.

3. Do *one (1)* of **a** or **b**. *[10]*

a. A fair coin is tossed until it comes up tails. This experiment is repeated independently three times, with the random variables X_1, X_2, and X_3 recording the number of tosses on the first, second, and third run of the experiment, respectively. Find the expected value and variance of $X = X_1 + X_2 + X_3$, as well as the probability function of X.

b. The continuous random variable Y has density function $g(y) = \begin{cases} ye^{-y} & y \geq 0 \\ 0 & y \leq 0 \end{cases}$. Show that $g(y)$ is a valid density function and find the expected value of Y.

[Total = 30]