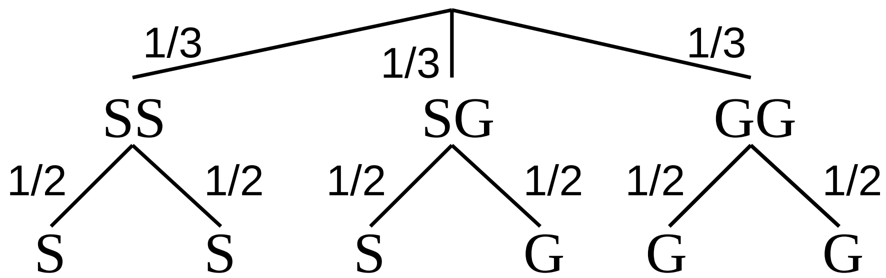
Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Winter 2018

Solution to Assignment #4
The pot at the end of the rainbow.

1. You reach the end of the rainbow – never mind how! – and a leprechaun offers you your choice of three identical-looking pots. The leprechaun tells you that one of the pots contains two gold coins, one contains two silver coins, and one contains a gold and a silver coin. You are not allowed to inspect the pots or their contents before making your choice, so you choose one of the pots at random. After being given the pot, you reach in and randomly take out one of the two coins in it, without looking at the other. If the coin you took out happens to be gold, what is the probability that the other coin in the pot is also gold? *[10]*

SOLUTION. For reference, here is the tree diagram of the complete experiment:



Initially, each of the three pots has an equal probability of $\frac{1}{3}$ of being chosen. Once a pot has been chosen, each coin in that pot has a probability of $\frac{1}{2}$ of being the one to be taken out. However, the information that the coin taken out is gold turns this into a conditional probability problem, Note that asking whether the other coin in the pot is gold is asking whether the chosen pot is the one with two gold coins. So we are really asking for the probability that the chosen pot is the one with two gold coins, given that the coin taken out was gold.

In the given experiment, let A be the event that the chosen pot is the one with two gold coins, and let B be the event that the coin taken out is gold. The pot with two gold coins has a probability of $\frac{1}{3}$ of being chosen, so $P(A) = \frac{1}{3}$. Each of the three gold coins in the experiment has a probability of $\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$ of becoming the coin to be taken out – see the tree diagram above – so $P(B) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$. Since choosing the pot with two gold coins means that the coin taken out must be gold, we see that $A \subset B$, so $A \cap B = A$. It follows that the probability that the chosen pot is the one with two gold coins, given that the coin taken out was gold, is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{1/3}{1/2} = \frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3} \quad \blacksquare$$

NOTE. This problem is a slightly dressed-up version (pots, leprechauns, rainbow :-) of what is nowadays called the *Bertrand's box paradox*, first given by Joseph Bertrand in 1889. It is somewhat similar in how it works to the more recent *Monty Hall problem*, which was first posed and solved by Steve Selvin in 1975 and later popularized by Marylin vos Savant in 1990.