Mathematics 1550H – Introduction to probability
Trent University, Winter 2018

Solutions to Assignment #3
Scramble

Consider the word "suspension".

1. How many different ways are there to arrange all the letters in the word if copies of the same letter can be told apart? *[2]*

Solution. "Suspension" has 10 letters, counting the duplicates. If these can all be told apart, there are 10! ways to arrange them. ■

2. How many different ways are there to arrange all the letters in the word if copies of the same letter are indistinguishable? *[4]*

Solution. The 10 letters of the word include 3 copies of "s" and 2 copies of "n". If copies of the same letter cannot be told apart, any rearrangement of these copies among themselves does not change an overall arrangement of all 10 letters. In any overall arrangment, here are 3! ways to rearrange the three copies of "s" among themselves and 2! ways to rearrange the two copies of "n" among themselves, so there are $\dfrac{10!}{3!2!}$ ways that can be told apart to arrange all 10 letters. ■

3. How many ways are there to arrange some, but not all, of the letters that appear in the word if at most one copy of each letter can be used? *[4]*

Solution. If we discard the duplicate letters in "suspension", we are left with 7 letters: "s", "u", "p", "e", "n", "i", and "o". Choosing "some, but not all" of these means picking 6, 5, 4, 3, 2, or 1 of them. (One could also legitimately include 0 here if one interprets "some" to include "none". This will increase the final answer below by 1 since there is only one way to choose and then arrange no letters.) There are $\binom{7}{k}$ ways to choose k letters out of 7 letters; once chosen there are $k!$ ways to arrange them. Adding these up gives:

$$
\begin{aligned}
&\binom{7}{6}\cdot 6! + \binom{7}{5}\cdot 5! + \binom{7}{4}\cdot 4! + \binom{7}{3}\cdot 3! + \binom{7}{2}\cdot 2! + \binom{7}{1}\cdot 1! \\
&= \frac{7!}{6!1!}\cdot 6! + \frac{7!}{5!2!}\cdot 5! + \frac{7!}{4!3!}\cdot 4! + \frac{7!}{3!4!}\cdot 3! + \frac{7!}{2!5!}\cdot 2! + \frac{7!}{1!6!}\cdot 1! \\
&= \frac{7!}{1!} + \frac{7!}{2!} + \frac{7!}{3!} + \frac{7!}{4!} + \frac{7!}{5!} + \frac{7!}{6!}
\end{aligned}
$$

You can work out the number! ■