Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Winter 2018
Solutions to Assignment #2
Tossing the coin away...

Consider the following experiment: Toss a fair coin repeatedly until a head immediately followed by a tail come up. For example, the shortest possible run of the experiment would be the outcome HT, the next shortest would be the outcomes HHT or THT, and so on.

1. What is the sample space for this experiment? *[2]*

SOLUTION. Here is the sample space:

$$\Omega = \{\, \text{HT, HHT, THT, HHHT, THHT, TTHT, HHHHT, THHHT, TTHHT, TTTHT}, \ldots \,\}$$

In general, the outcomes in the sample space look like a string (possibly empty) of Ts, followed by a string of at least one H, followed by a T. ∎

2. What is the probability function for this experiment? *[4]*

SOLUTION. Since the coin is fair, each outcome has probability $\left(\frac{1}{2}\right)^n$, where n is the length of the outcome. To be more explicit, if the outcome is $\omega = \text{T}^k\text{H}^\ell T$, *i.e.* k Ts followed by ℓ Hs and a T, then $m(\omega) = \left(\frac{1}{2}\right)^{k+\ell+1}$. ∎

3. What is the probability that at least five tosses will occur during this experiment? *[4]*

SOLUTION. This is best done slightly indirectly:

$$P(\text{at least 5 tosses}) = 1 - P(\text{less than 5 tosses}) = 1 - P(\leq 4 \text{ tosses})$$
$$= 1 - [m(\text{HT}) + m(\text{HHT}) + m(\text{THT}) + m(\text{HHHT}) + m(\text{THHT}) + m(\text{TTHT})]$$
$$= 1 - \left[\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}\right] = 1 - \frac{11}{16} = \frac{5}{16} \quad \blacksquare$$