Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Winter 2018

Solution to Assignment #1
Is it random?

Consider the following two sequences of one hundred heads and tails:

Sequence #1:

TTHHHTHTHHHTTHHTTTTHHHTHHTHHTHTTTHTHTTHTHHTHTTHTTTHTHHHHHTTTHHTHTHHHHHTHTHHTHTTTTTHHHHHTHHHTTTHTTHTT

Sequence #2:

TTHHTTTHTHTHTTTTHHTHHTTTHHHHTHHHTTHTHHTHTHTHTTHTTTHHHTHTHTHTHTTTHTTHHHTHHTHTTHHHTTHTTTHHHTHHTTHTTHTH

One of these sequences was generated by actually tossing a quarter one hundred times; the other was generated by your instructor sitting at his computer and hitting the "H" and "T" keys one hundred times between them and trying to make it seem random.

1. Try to figure out which sequence was generated by tossing a coin and which was not. Give your reasoning! *[10]*

SOLUTION. Sequence #1 is the sequence that was generated by actually tossing a coin. There are a number of ways one might try to distinguish it from the not-truly-random sequence #2, but the easiest is to observe that sequence #1 has several runs all `H`s or all `T`s of length five, whereas sequence #2 has no runs that long. It turns out that if you toss a coin one hundred times, it is very likely that you will have one or more runs of length five or more. [This is the first time I've done this experiment, by the way, that did not have a run of six or more.] It is not as likely that a human trying to generate a random-seeming sequence will have many long runs, if any, because these do not *feel* random. ■