**Mathematics 1550H – Introduction to probability**
TRENT UNIVERSITY, Winter 2018

FINAL EXAMINATION
*Tuesday, 17 April, 2018*

**Spatio-temporal locus:** 19:00–22:00 in the Gym *Inflicted by* Стефан Біланюк.

**Instructions:** Do both of parts **Bernoulli** and **Chebyshev**, and, if you wish, part **Dopey**. Show all your work and simplify answers as much as practical. *If in doubt about something,* **ask!**

**Aids:** Calculator; one $8.5'' \times 11''$ or A4 aid sheet; standard normal table; lots of neurons.

**Part Bernoulli.** Do all of **1**–**5**. *[Subtotal = 68/100]*

**1.** A fair six-sided non-standard die has faces numbered 0, 1, 1, 2, 2, and 2, respectively. The random variable $X$ records the number that comes up on a single roll of the die.

**a.** What is the probability function of $X$? *[5]*

**b.** Compute the expected value $E(X)$ and variance $V(X)$ of $X$. *[5]*

**2.** Let $T$ be a continuous random variable with the following probability density function:

$$f(t) = \begin{cases} |t| & -1 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

**a.** Verify that $f(t)$ is indeed a probability density function. *[8]*

**b.** If you know that $T \leq \frac{1}{2}$, what is the probability that $T \geq 0$? *[7]*

**3.** A fair coin is tossed until it comes up heads, then tossed some more until it comes up tails. The random variable $Y$ counts the total number of times the coin is tossed during the experiment.

**a.** Find the probability function, expected value, and variance of $Y$. *[12]*

**b.** Use Chebyshev's Inequality to estimate the probability that $Y \geq 9$. *[5]*

**c.** Compute the probability that $Y \geq 9$. *[5]*

**4.** A hand of five cards is randomly chosen, simultaneously and without replacement, from a standard 52-card deck.

**a.** What is the probability that the hand includes all four of one kind*? *[5]*

**b.** What is the probability that the hand includes at least three of one kind? *[5]*

**c.** What is the probability that the hand includes four of one kind, given that it includes at least three of one kind? *[5]*

**5.** Suppose $U$ is a continuous random variable that has a normal distribution with expected value $\mu = -2$ and variance $\sigma^2 = 1$. Compute $P(-1.1 \leq U \leq 1.1)$ with the help of a standard normal table. *[6]*

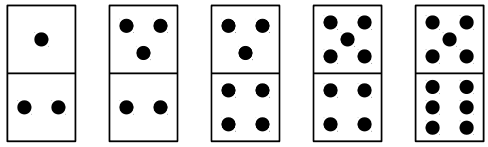
*[Parts* **Chebyshev** *and* **Dopey** *are on page 2.]*

* Recall that the *kinds* are $A$, $K$, $Q$, $J$, 10, 9, 8, 7, 6, 5, 4, 3, and 2. The *suits* are ♡, ♢, ♣, and ♠.

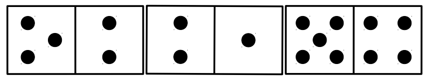
**Part Chebyshev.** Do any *two* (2) of **6–9**. *[Subtotal = 32/100]*

**6.** You are given five dominoes, marked as in the figure at right.

The five dominoes:



Three dominoes end-to-end, with one pair of adjacent ends matching:



**a.** How many ways are there to choose three of the dominoes and lay them out end-to-end? *[8]*

**b.** If three of the dominoes are chosen at random and laid out end-to-end randomly, what is the probability that both pairs of adjacent ends will match? *[8]*

**7.** A frog and a toad have a race of sorts from one end of a straight 3 $m$ track to the other. The frog hops 10 $cm$ at a time and the toad hops 15 $cm$ at a time, and they both head straight from the starting line to the finish line. However, the frog and toad do not move all the time. Every 2 seconds the frog, with equal probability, either makes a hop toward the finish line or stays still; every 3 seconds the toad, with equal probability, either makes a hop toward the finish line or stays still. Assuming they begin at the starting line at the same time, which would you expect to reach the finish line first? *[16]*

**8.** Suppose $X_1$ and $X_2$ are independent continuous random variables that each have an exponential distribution with $\lambda = 1$. Let $X = X_1 + X_2$.

**a.** Compute the expected value, $E(X)$, and variance, $V(X)$, of $X$. *[6]*

**b.** Find the probability density function of $X$. *[10]*

**9.** Suppose the discrete random variables $X$ and $Y$ are jointly distributed according to the following table:

| $Y \backslash X$ | 2 | 3 | 4 |
|---|---|---|---|
| $-1$ | 0.1 | 0.2 | 0.2 |
| 0 | 0.2 | 0 | 0 |
| 1 | 0.2 | 0 | 0.1 |

**a.** Compute the expected values $E(X)$ and $E(Y)$, the variances $V(X)$ and $V(Y)$, and also the covariance $\mathrm{Cov}(X, Y)$ of $X$ and $Y$. *[10]*

**b.** Determine whether $X$ and $Y$ are independent. *[2]*

**c.** Let $W = 2X + Y$. Compute $E(W)$ and $V(W)$. *[4]*

*[Total = 100]*

**Part Dopey.** Bonus!

• . Two fair standard dice are rolled simultaneously three times. What is the probability that they will come up with the same face on at least one of the three rolls? *[1]*

⠿ . Write a haiku touching on probability or mathematics in general. *[1]*

**haiku?**

seventeen in three:
five and seven and five of
syllables in lines

*[Part* **Bernoulli** *is on page 1.]*

I HOPE THAT YOU ENJOYED THE COURSE. HAVE A GOOD SUMMER!