

Mathematics 1550H – Introduction to probability

TRENT UNIVERSITY, Winter 2018

Assignment # 11

Random Walks the Plane

Due on Friday, 6 April.

Random decides to take a walk in the Cartesian plane, starting at the origin $(0, 0)$. Random being random, the next step is always decided by simultaneously rolling two fair standard dice, a red one and a blue one. If the blue one comes up 1 or 2, move left one unit; if it comes up 3 or 4 don't move to either side; if it comes up 5 or 6, move right one unit. At the same time, if the red one comes up 1 or 2, move down by 1; if it comes up 3 or 4, don't move up or down; if it comes up 5 or 6, move up one unit. For example, if Random is at the point $(3, 1)$ and rolls 1 on the blue die and 4 on the red die, Random will step to the point $(3 - 1, 1 + 0) = (2, 1)$.

Let (X_n, Y_n) denote Random's position after n steps. Of course, $(X_0, Y_0) = (0, 0)$, but after that chance rules.

1. Find the expected values and variances of X_n and Y_n . [5]
2. Find, as best you can, the expected value and variance of the distance that Random is from the origin after n steps. [5]