Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Winter 2018

Assignment # 10
Chebyshev's (In)equality
Due on Wednesday, 4 April.

The version of Chebyshev's Inequality we saw in class states that a random variable X with expected value $\mu = E(X)$ and standard deviation $\sigma = \sqrt{V(X)}$ must satisfy

$$P\left(|X - \mu| \geq \varepsilon\right) \leq \frac{\sigma^2}{\varepsilon^2}$$

for any real number $\varepsilon > 0$. A common variation of Chebyshev's Inequality states that a random variable X with expected value $\mu = E(X)$ and standard deviation $\sigma = \sqrt{V(X)}$ must satisfy

$$P\left(|X - \mu| \geq k\sigma\right) \leq \frac{1}{k^2}$$

for any real number $k > 0$. The two forms are equivalent – let $\varepsilon = k\sigma$ to see why – but the latter is often more convenient in statistics, where measuring the distance between X and μ in terms of σ is likely to be pretty natural. Note that the second form is of interest only if $k > 1$, since if $k \leq 1$, then $\frac{1}{k^2} \geq 1$ and any probability must be ≤ 1.

1. Given a fixed real number $k > 1$, any fixed real number μ, and any fixed real number $\sigma > 0$, find a probability density function $g(x)$ such that a continuous random variable X with this density will have $E(X) = \mu$, standard deviation $\sqrt{V(X)} = \sigma$, and satisfy $P\left(|X - \mu| \geq k\sigma\right) = \frac{1}{k^2}$. *[10]*

NOTE: The point here is that Chebyshev's Inequality can't really be improved upon unless you have additional information about the distribution of the random variable X.