TRENT UNIVERSITY, WINTER 2017

**MATH 1550H Test**

Thursday, 2 March, 2017

*Time: 50 minutes*

**Instructions**

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet.

**1.** Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

**a.** Suppose the continuous random variable $X$ has $f(x) = \begin{cases} 1+x & -1 \le x \le 0 \\ 1-x & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$ as its probability density function. Compute $P(-0.5 \le X \le 0.5)$.

**b.** A hand of five cards is drawn simultaneously and randomly from a standard 52-card deck. What is the probability that the hand includes exactly three ♡s?

**c.** A fair coin is tossed four times. What is the probability that at least two heads will come up?

**2.** Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

**a.** Show that if $A$ and $B$ are events in some sample space, with $P(A) > 0$ and $P(B) > 0$, then $\dfrac{P(A|B)}{P(A)} = \dfrac{P(B|A)}{P(B)}$.

**b.** Determine whether $g(x) = \begin{cases} x^{-2} & 1 \le x \\ 0 & x < 1 \end{cases}$ is a valid continuous probability density.

**c.** A fair non-standard six-sided die thas one face numbered 1, two faces numbered 2, and three faces numbered 3. What is the expected value of the number that comes up if the die is rolled once?

**3.** Do *one (1)* of **a** or **b**. *[10]*

**a.** The continuous random variable $X$ has an exponential distribution with $\lambda = 1$. Let $A$ be the event that $X \le \ln(3)$ and $B$ be the event that $\ln(2) \le X \le \ln(4)$. Determine whether $A$ and $B$ are independent or not. [Recall that $e^{\ln(t)} = t$ for all $t > 0$ and that $\ln(a) < \ln(b)$ whenever $0 < a < b$.]

**b.** A fair coin is tossed once and then tossed again until it comes up with the same face that came up on on the first toss. Let the random variable $Y$ count the total number of tosses that occur in this experiment. Find the probability funtion of $Y$ and compute the expected value, $E(Y)$, of $Y$.

*[Total = 30]*