Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Winter 20175

Assignment #1
(Un)biased?
Due on Thursday, 26 February, 2017.

1. Suppose all you have is a biased coin that when tossed comes up heads 41% of the time, tails 58% of the time, and lands on edge 1% of the time. How can you use it to simulate a fair coin, *i.e.* one that comes up heads 50% of the time and tails 50% of the time? Explain why your method works. *[3]*

2. Suppose all you have is a fair coin. How can you use it to simulate a biased coin such as the one in problem **1** that comes up heads 41% of the time, tails 58% of the time, and lands on edge 1% of the time? Explain why your method works. *[2]*

3. Suppose all you have is a coin with an unknown bias, but which has a thin enough edge so that it always lands either head or tails when tossed, and will land on each of heads and tails some of the time. How can you use it to simulate a biased coin such as the one in problem **1** that comes up heads 41% of the time, tails 58% of the time, and lands on edge 1% of the time? Explain why your method works. *[2]*

4. Suppose all you have is a fair coin. How can you use it to – completely accurately! – simulate a biased coin that has $P(H) = \frac{1}{\pi}$ and $P(T) = 1 - \frac{1}{\pi}$? Explain why your method works, or explain why there can be no such method. *[3]*

Note: Keep in mind that $\frac{1}{\pi}$ is irrational (because π is, and so cannot be written as a ratio of integers. That makes this problem harder than **2** and will probably require a different method, if there is any to be had ...