Mathematics 1550H – Introduction to probability

TRENT UNIVERSITY, Winter 2016

MATH 1550H Test

Monday, 22 February, 2016
Time: 50 minutes

Instructions
- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet.

1. Do any *three (3)* of **a–d**. *[12 = 3 × 4 each]*

a. The continuous random variable W has the density function $f(t) = \begin{cases} \frac{1}{4} & -2 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$.
Compute $P(W > 1)$.

b. How many distinct ways are there to arrange ten books, three of which are identical to one another, on four shelves? (Each shelf could accommodate all ten books.)

c. A five-card hand is drawn at random from a standard 52-card deck. What is the probability that each of the five cards is of a different kind?

d. A fair five-sided die with faces numbered 1 to 5 is rolled twice. Let X be the sum of the faces that come up on the two rolls. Find the probability function of X.

2. Do any *two (2)* of **a–c**. *[10 = 2 × 5 each]*

a. If A and B are events in a sample space Ω, does $P(A|B) + P(A|\bar{B}) = P(A)$? Verify that it must be so or find an example demonstrating otherwise.

b. A fair coin is tossed five times. Let A be the event that exactly two heads occurred in the five tosses and B be the event that the first two tosses included one head and one tail. Determine whether A and B are independent or not.

c. The continuous random variable X has density function $g(t) = \begin{cases} 1 - \frac{1}{2}t & 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$.
Find the *median* of X; that is, the number m such that $P(X \le m) = \frac{1}{2}$.

3. Do any *one (1)* of **a** or **b**. *[8 = 1 × 8 each]*

a. Suppose the continuous random variable X has density function $g(t) = \begin{cases} e^{-t} & 0 \le t \\ 0 & t < 0 \end{cases}$.
Let A be the event that $X > 2$ and B be the event that $X > 1$. Compute $P(A|B)$.

b. A hand of four cards is randomly chosen, without replacement, from a standard 52-card deck. What is the probability that at least one suit does not occur among the four cards?

[Total = 30]