Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Winter 2016

Assignment #3
Irrational Bias
Due on Friday, 11 March, 2016.

You die[†]. For your transgressions while still alive[*] you are initially placed by yourself in a featureless room with just one object: a thick biased coin which has a probability, when tossed, of getting a head of $P(H) = \frac{1}{\pi}$, a probability of getting a tail of $P(T) = \frac{1}{\sqrt{3}}$, and a probability of landing on edge of $P(E) = 1 - \frac{1}{\pi} - \frac{1}{\sqrt{3}}$. The Highest Authority gives you the following problems to solve[‡], with the promise that if and when you solve them, you can move on to the rest of your afterlife.

1. How could you simulate a fair coin using the biased coin you have been given? *[2]*

2. How could you simulate a fair standard six-sided die using the given coin? *[2]*

3. How could you simulate a biased coin with $P(H) = \frac{3}{5} = 0.6$ and $P(T) = \frac{2}{5} = 0.4$ using the given coin? *[2]*

4. How could you simulate a biased coin with $P(H) = \frac{1}{\sqrt{2}}$ and $P(T) = 1 - \frac{1}{\sqrt{2}}$ using the given coin? *[4]*

NOTE: $\frac{1}{\pi}$, $\frac{1}{\sqrt{3}}$, and $1 - \frac{1}{\pi} - \frac{1}{\sqrt{3}}$, as well as $\frac{1}{\sqrt{2}}$ and $1 - \frac{1}{\sqrt{2}}$, are all irrational, and so cannot be expressed as ratios of integers. Also, their decimal expansions (and expansions in other bases) are infinite and non-repeating.

[†] Did you divide by zero? Noooooooo ...

[*] If you don't have worthy transgressions, like dividing by zero, just imagine that you did.

[‡] No one expects to meet the Mathematical Inquisition once they're dead!