Trent University, Summer 2023 (S62)

MATH 1550H Test

Monday, 10 July

Time: 50 minutes

Name: ______________________________

Student Number: ______________________________

Question	**Mark**	
1	____	
2	____	
3	____	
Total	____	/30

Instructions

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- If you do more parts of a question than were asked for, cross out the one you do not want marked; otherwise the marker will not consider the part they encounter last.
- You may use a calculator, so long as it cannot communicate with other devices, and one letter- or A4-size aid sheet with whatever you want written on all sides of it.

1. Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

a. How many different ways are there to arrange all the letters in the word "unusual" if the three copies of "u" cannot be told apart?

b. A biased coin with $P(H) = 0.6$ and $P(T) = 0.4$ is tossed three times. What are the sample space and probability distribution function for this experiment?

c. A five-card hand is drawn from a standard 52-card deck, without order or replacement. What is the probability that the cards in the hand are all of different kinds (*i.e.* ranks)?

2. Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

a. The continuous random variable X has $f(x) = \begin{cases} x/8 & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$ as its probability density function. Compute $P(X \geq 1 \mid X \leq 3)$.

b. A biased coin with $P(H) = 0.6$ and $P(T) = 0.4$ is tossed three times. The random variable Y counts the number of heads that come up in those three tosses. Compute the expected value of Y.

c. Suppose $g(x) = \begin{cases} c\left(1 - x^2\right) & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$, where c is a constant. Find the value of the constant c which would make $g(x)$ a valid probability density function.

3. Do *one (1)* of **a** or **b**. *[10]*

a. Suppose A and B are independent events in some sample space S, with both having positive probability. Does it have to be the case that $\bar{A} = \{\, s \in S \mid s \notin A \,\}$ is independent of B? If so, explain why; if not, give an example in which A and B are independent, but $\bar{A}$ and B are not.

b. A doctor gives a patient a test for a condition which occurs in the 1% of the population. Experience has shown that the test returns positive in 99% of the time and negative 1% of the time if the condition is present, and returns positive 5% of the time and negative 95% of the time if the condition is not present. If the test given by the doctor comes back positive, how likely is it that the patient actually has condition A?

[Total = 30]