

# Mathematics 1550H – Probability I: Introduction to Probability

TRENT UNIVERSITY, Summer 2023 (S62)

## Quiz #5

### Shuffling the Deck

*Due\* just before midnight on Thursday, 6 July.*

**Instructions:** Do the following problem. Please show all your work.

1. You shuffle a standard deck thoroughly to get a random arrangement of the 52 cards. Is it likely that this arrangement has been achieved by someone shuffling a deck before? Explain why or why not. [5]

SOLUTION. There are  $52!$  ways of arranging the 52 cards in a standard deck. This is a pretty big number:

$$\begin{aligned} 52! &= 8065817517094387857166063685640376697528950544088327782400000000000 \\ &= 8.0658175170943878571660636856403766975289505440883277824 \times 10^{67} \end{aligned}$$

Let's compare it to what is (probably :- ) an overestimate of the number of arrangements achieved by shuffling a standard deck in history. The standard 52 card deck in something like its modern form seems to have been developed by about 1480 A.D., less than 600 years ago. Just to be on the safe side, and also for ease of calculation, let's suppose that it has been around for 1000 years. There are 365 or 366 days in each (Julian or Gregorian) calendar year; in either case, less than 1000 days per year, which is the number we'll use, again for ease of calculation. Let's suppose that on each day a trillion, *i.e.*  $1000000000000 = 10^{12}$  shuffles of a full deck occurred, each resulting in a different arrangement. (That would be something like 100 shuffles every day by every person in a population of 10 billion people . . . ) This would result in

$$1000 \times 1000 \times 1000000000000 = 10^3 \cdot 10^3 \cdot 10^{12} = 1.0 \times 10^{18}$$

possible different arrangements.

This probable overestimate of the number of arrangements of a standard deck previously achieved by someone shuffling away is vanishingly small compared to the number of possible arrangements, so it's pretty likely that any thorough shuffle of the standard deck is a new arrangement.  $\square$

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\* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If this fails, you may submit your work to the instructor on paper or by email to [sbilaniuk@trentu.ca](mailto:sbilaniuk@trentu.ca).