

## Mathematics 1550H – Probability I: Introduction to Probability

TRENT UNIVERSITY, Summer 2023 (S62)

### Quiz #3

#### Conditional Probability and Independence

*Due\* just before midnight on Thursday, 29 June.*

**Instructions:** Do any *two* (2) of the following three problems. (If you do all three, only the first two encountered by the marker will get marked.) Please show all your work.

1. Suppose  $S$  is a sample space and  $A$ ,  $B$ , and  $C$  are events in  $S$ , with  $P(A) > 0$ ,  $P(B) > 0$ , and  $P(C) > 0$ , such that  $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$ . Does it follow that  $A$  and  $B$  are independent? [2.5]

NOTE: You should either explain – correctly! – why  $A$  and  $B$  must be independent, or give an example of a sample space  $S$  and events  $A$ ,  $B$ , and  $C$  which meet the above conditions but for which  $A$  and  $B$  are dependent.

2. You are given 65 coins, 64 of which are fair and 1 of which has two heads. One coin is selected at random from the 65, and then tossed 6 times, coming up heads on every toss. What is the probability that the selected coin is the two-headed one? [2.5]
3. A box contains three different types of disposable flashlights. Suppose that 20% of the flashlights in the box are of type  $A$ , 30% are of type  $B$ , and 50% are of type  $C$ . The probabilities that type  $A$ , type  $B$ , and type  $C$  flashlights will last over 100 hours of use are respectively 0.70, 0.40, and 0.30.
  - a. What is the probability that a flashlight randomly chosen from the box will last over 100 hours of use? [1]
  - b. If a flashlight randomly chosen from the box lasted over 100 hours, what is the probability it was of type  $B$ ? [1.5]

---

\* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If this fails, you may submit your work to the instructor on paper or by email to [sbilaniuk@trentu.ca](mailto:sbilaniuk@trentu.ca).