Quiz #11
A Joint Distribution

Instructions: Do the following problem. Please show all your work.

1. Suppose the discrete random variables X and Y are jointly distributed according to the given table.

   a. Compute the expected values $E(X)$ and $E(Y)$, the variances $V(X)$ and $V(Y)$, and also the covariance $\text{Cov}(X,Y)$ of X and Y. *[3.5]*

   b. Determine whether X and Y are independent. *[0.5]*

   c. Let $T = X - Y$. Compute $E(T)$ and $V(T)$. *[1]*

$Y \backslash X$	0	1	2
1	0.2	0.2	0.2
2	0.2	0	0.1
3	0	0.1	0

SOLUTIONS. **a.** Here we go:

$$E(X) = 0(0.2 + 0.2 + 0) + 1(0.2 + 0 + 0.1) + 2(0.2 + 0.1 + 0)$$
$$= 0 \cdot 0.4 + 1 \cdot 0.3 + 2 \cdot 0.3 = 0 + 0.3 + 0.6 = 0.9$$
$$E(Y) = 1(0.2 + 0.2 + 0.2) + 2(0.2 + 0 + 0.1) + 3(0 + 0.1 + 0)$$
$$= 1 \cdot 0.6 + 2 \cdot 0.3 + 3 \cdot 0.1 = 0.6 + 0.6 + 0.3 = 1.5$$
$$E\left(X^2\right) = 0^2(0.2 + 0.2 + 0) + 1^2(0.2 + 0 + 0.1) + 2^2(0.2 + 0.1 + 0)$$
$$= 0 \cdot 0.4 + 1 \cdot 0.3 + 4 \cdot 0.3 = 0 + 0.3 + 1.2 = 1.5$$
$$E\left(Y^2\right) = 1^2(0.2 + 0.2 + 0.2) + 2^2(0.2 + 0 + 0.1) + 3^2(0 + 0.1 + 0)$$
$$= 1 \cdot 0.6 + 4 \cdot 0.3 + 9 \cdot 0.1 = 0.6 + 1.2 + 0.9 = 2.7$$
$$V(X) = E\left(X^2\right) - [E(X)]^2 = 1.5 - 0.9^2 = 1.5 - 0.81 = 0.69$$
$$V(Y) = E\left(Y^2\right) - [E(Y)]^2 = 2.7 - 1.5^2 = 2.7 - 2.25 = 0.45$$
$$E(XY) = 1 \cdot 0 \cdot 0.2 + 1 \cdot 1 \cdot 0.2 + 1 \cdot 2 \cdot 0.2$$
$$+ 2 \cdot 0 \cdot 0.2 + 2 \cdot 1 \cdot 0 + 2 \cdot 2 \cdot 0.1$$
$$+ 3 \cdot 0 \cdot 0 + 3 \cdot 1 \cdot 0.1 + 3 \cdot 2 \cdot 0$$
$$= 0 + 0.2 + 0.4 + 0 + 0 + 0.4 + 0 + 0.3 + 0 = 1.3$$
$$\text{Cov}(X,Y) = E(XY) - E(X)E(Y) = 1.3 - 0.9 \cdot 1.5 = 1.3 - 1.35 = -0.05 \quad \square$$

b. Since $\text{Cov}(X,Y) = -0.05 \neq 0$, X and Y are not independent. $\square$

c. Expected value is easy: $E(T) = E(X - Y) = E(X) - E(Y) = 0.9 - 1.5 = -0.6$. We have to work a little harder for variance:

$$V(T) = V(X - Y) = V(X + (-1)Y) = V(X) + V((-1)Y) + 2\,\text{Cov}\,(X, (-1)Y)$$
$$= V(X) + (-1)^2 V(Y) + 2(-1)\text{Cov}(X,Y) = 0.69 + 0.45 - 2 \cdot (-0.05)$$
$$= 01.14 - (-0.1) = 1.14 + 0.1 = 1.24 \quad \blacksquare$$