**Mathematics 1550H – Probability I: Introduction to Probability**
TRENT UNIVERSITY, Summer 2023 (S62)

**Solutions to Quiz #1**
**Alea iacta est.**[†]
*Due*[*] *just before midnight on Thursday, 22 June.*

A fair six-sided die with faces numbered 1 through 6 is rolled twice and the number that comes up on each roll is recorded.

**1.** What is the sample space for this experiment? *[1]*

SOLUTION. The sample space consists of all possible pairs of faces, *i.e.* pairs of integers where each of the integers is one of the numbers 1 through 6. That is, the sample space $S$ for this experiment is, explicitly:

$$\begin{aligned} S = \{\, &(1,1),\ (1,2),\ (1,3).\ (1,4),\ (1,5),\ (1,6),\\ &(2,1),\ (2,2),\ (2,3).\ (2,4),\ (2,5),\ (2,6),\\ &(3,1),\ (3,2),\ (3,3).\ (3,4),\ (3,5),\ (3,6),\\ &(4,1),\ (4,2),\ (4,3).\ (4,4),\ (4,5),\ (4,6),\\ &(5,1),\ (5,2),\ (5,3).\ (5,4),\ (5,5),\ (5,6),\\ &(6,1),\ (6,2),\ (6,3).\ (6,4),\ (6,5),\ (6,6)\,\}\\ = \{\, &(a,b) \mid 1 \le a \le 6 \text{ and } 1 \le b \le 6\,\} \quad \square \end{aligned}$$

**2.** What is the probability function for this experiment? *[1]*

SOLUTION. The die is fair, so every face has an equal chance of coming up on any given roll. It follows that every pair of faces has an equal chance of coming up when the die is rolled twice. Since there are 36 possible pairs of faces and their probabilities must add up to 1, it follows that $P(a,b) = \dfrac{1}{36}$ for every pair $(a,b) \in S$. $\square$

**3.** What are the probabilities of each of the following possible events for this experiment?

**a.** The sum of the numbers that come up on the two rolls is 8. *[1]*

**b.** The difference between the numbers that come up on the first and second rolls has absolute value $\le 1$. *[1]*

**c.** The number that came up on the first roll is odd and the number that came up on the second role is even. *[1]*

---

[†] "The die is cast." Attributed to Gaius Julius Caesar when his army crossed the Rubicon river from Cisalpine Gaul into Roman Italy proper, setting off a civil war.

[*] You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If this fails, you may submit your work to the instructor on paper or by email to `sbilaniuk@ trentu.ca` .

SOLUTIONS. **a.** Let $E$ be the event that the numbers that come up on the two rolls add up to 8. Then $E = \{\,(2,6),\,(3,5),\,(4,4),\,(5,3),\,(6,2)\,\}$, so the probability of $E$ is:

$$\begin{aligned} P(E) &= P(2,6) + P(3,5) + P(4,4) + P(5,3) + P(6,2) \\ &= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{5}{36} \approx 0.1389 \quad \square \end{aligned}$$

**b.** Let $D$ be the event that the difference between the numbers that come up on the first and second rolls has absolute value $\leq 1$. Then

$$\begin{aligned} D = \{\,&(1,1),\,(1,2),\,(2,1),\,(2,2),\,(2,3),\,(3,2),\,(3,3),\,(3,4),\\ &(4,3),\,(4,4),\,(4,5),\,(5,4),\,(5,5),\,(5,6),\,(6,5),\,(6,6)\,\}\ , \end{aligned}$$

so, since every pair in $D$ has a probability of $\dfrac{1}{36}$ of coming up, the probability of $D$ is:

$$P(D) = \frac{\#\text{ of pairs in } D}{\#\text{ of pairs in } S} = \frac{16}{36} = \frac{4}{9} \approx 0.4444 \quad \square$$

**c.** Let $C$ be the event that the number that came up on the first roll is odd and the number that came up on the second role is even. Then

$$C = \{\,(1,2),\,(1,4),\,(1,6),\,(3,2),\,(3,4),\,(3,6),\,(5,2),\,(5,4),\,(5,6)\,\}\ ,$$

so, since every pair in $C$ has a probability of $\dfrac{1}{36}$ of coming up, the probability of $C$ is:

$$P(C) = \frac{\#\text{ of pairs in } C}{\#\text{ of pairs in } S} = \frac{9}{36} = \frac{1}{4} = 0.25 \quad \square$$