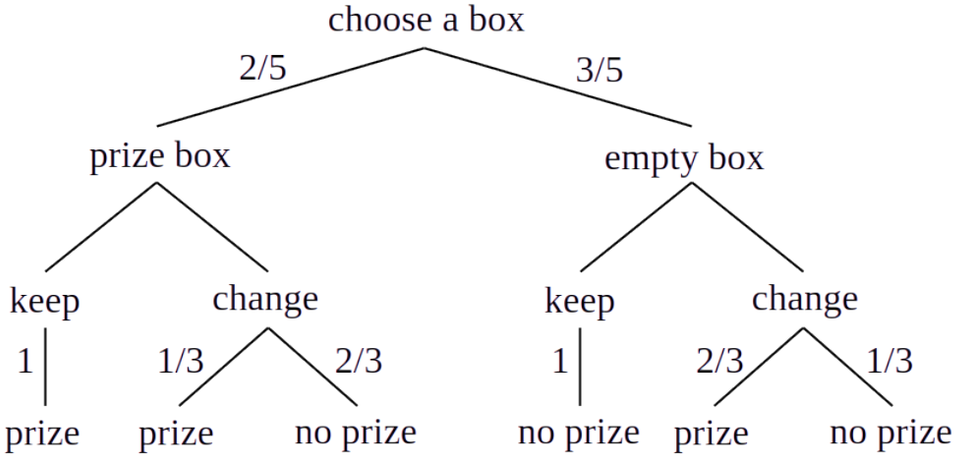
Mathematics 1550H – Probability I: Introduction to Probability
TRENT UNIVERSITY, Summer 2023 (S62)

Assignment #3
Switcheroo!

1. Playing a game at a carnival, you are asked to choose one of five boxes, which are identical in appearance. Two of the boxes contain a prize and the other three are empty. Once you select a box, the carnie running the game opens one of the other boxes, shows you that it is empty, and gives you the chance to change your mind, *i.e.* to pick an unopened box other than the one you selected first. Will changing your selection improve your chances of winning or not? Give a complete explanation! *[10]*

NOTE: This puzzle is a somewhat souped-up variation of the Monty Hall Problem, which you should feel free to look up.

SOLUTION. Here is a tree diagram of choices and probabilities for this problem, with explanation and application following.



When you initially choose one of the five boxes, you have a probability of $\frac{2}{5} = 0.4$ of choosing a box containing a prize and a probability of $\frac{3}{5} = 0.6$ of choosing an empty box. The carnie then opens one of the boxes you did not select and shows that it is empty, and gives you the option of selecting one of the unopended boxes that you did not select initially instead of the one you did.

In the case where your initial selection has a prize, the remaining three unopened boxes include two that are empty and one that has a prize. You are guaranteed a prize if you keep the box that you selected first, but if you change your mind, you have a probability of $\frac{1}{3}$ of choosing the unopened box with a prize and a $\frac{2}{3}$ chance of choosing an empty box.

In the case where your initial selection is empty, the remaining three unopened boxes include two that have a prize and one that is empty. You are guaranteed not to get a prize if you keep the box that you selected first, but if you change your mind, you have a probability of $\frac{2}{3}$ of choosing an unopened box with a prize and a $\frac{1}{3}$ chance of choosing the remaining empty box.

Of course, when the carnie offers you the choice of keeping the box you selected initially or choosing another, you do not know which case you are in. If you decide to keep the box

you initially chose, your total probability of getting a prize is $\frac{2}{5} \cdot 1 + \frac{3}{5} \cdot 0 = \frac{2}{5} = 0.4$. On the other hand, if you decide to choose one of the other unopened boxes, the total probability that you will get a prize is $\frac{2}{5} \cdot \frac{1}{3} + \frac{3}{5} \cdot \frac{2}{3} = \frac{2}{15} + \frac{6}{15} = \frac{8}{15} \approx 0.5333$.

Thus, given that you don't know which case you're in when the carnie makes the offer, your chances of getting a prize are better if you select one of the other unopened boxes. □