Mathematics 1550H – Probability I: Introduction to Probability
TRENT UNIVERSITY, Summer 2023 (S62)

Assignment #1
Simulations

Due just before midnight on Friday, 23 June.*

Something that comes up in a lot of practical applications of probability is simulating some desired random process by the one you actually have access to. The classic problem along these lines is wanting to have a fair coin, but only having a biased coin with even the bias not being known†. One can work around this by using the following process to simulate a fair coin:

1. Toss the biased coin twice.
2. If the outcome of the two coin tosses if step 1 is HH or TT, repeat step 1. Otherwise, return H for the simulated coin if the outcome of step 1 was HT, and return T for the simulated coin if the outcome of step 1 was TH.

Why does this process work? As long as the outcomes HH and TT of the biased coin have a combined probability less than 1, the probability that you will get HH and/or TT forever is zero, so the process will eventually terminate and return some outcome for the simulated fair coin. The two simulated outcomes are equally likely because however the coin being used is biased, the outcomes HT and TH are equally likely. (Why?)

In this assignment you will be asked to use some given random process or tool to simulate another one.

1. Explain how to use a fair standard six-sided die to simulate a fair coin. *[1]*

2. Explain how to use a fair coin to simulate a fair standard six-sided die. *[3]*

3. Explain how to use a fair coin to simulated a biased coin which has a probability, on any given toss, of $\frac{1}{3}$ of the head coming, of $\frac{1}{2}$ of the tail coming up, and $\frac{1}{6}$ of landing (and staying!) on its edge. *[3]*

4. Explain how to use a fair ten-sided die, with its faces numbered 0 through 9, to simulate a biased coin that has a probability of $\frac{1}{\pi}$ of coming up heads and a probability of $1 - \frac{1}{\pi}$ of coming up tails. *[3]*

NOTE. In question **4**, please keep in mind that, like $\sqrt{2}$, $\frac{1}{\pi}$ is an irrational number, *i.e.* it not equal to $\frac{a}{b}$ for any integers a and b.

* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If this fails, you may submit your work to the instructor on paper or by email to `sbilaniuk@ trentu.ca` .

† A real-world version of this problem, faced by most operating systems, is using the output of some hardware random number generator, which is likely to have some bias, to generate unbiased random numbers.