

Mathematics 1550H – Probability I: Introduction to Probability

TRENT UNIVERSITY, Summer 2020

Some Common Probability Distributions – The Short Form*

Discrete Distributions

1. *Discrete Uniform.* n equally likely outcomes for some $n \geq 1$.

$$\text{Probability function: } m(\omega) = \frac{1}{n}.$$

Expected value and variance of a random variable X on Ω depend on just what values X assigns to each outcome $\omega \in \Omega$.

2. *Bernoulli Trial.* Two outcomes with probability of success p and of failure $q = 1 - p$. X counts successes.

$$\text{Probability function: } m(1) = P(\text{success}) = p \text{ and } m(0) = P(\text{failure}) = q .$$

$$\text{Expected value: } \mu = E(X) = p \quad \text{Variance: } \sigma^2 = V(X) = pq$$

3. *Binomial.* n Bernoulli trials, with probability of success p and of failure $q = 1 - p$. X counts successes.

$$\text{Probability function: } m(k) = P(k \text{ successes}) = \binom{n}{k} p^k q^{n-k}, \text{ where } 0 \leq k \leq n.$$

$$\text{Expected value: } \mu = E(X) = np \quad \text{Variance: } \sigma^2 = V(X) = npq$$

4. *Geometric.* Bernoulli trials repeated until the first success, with probability of success p and of failure $q = 1 - p$. X counts the number of trials required.

$$\text{Probability function: } m(k) = P(\text{first success on } k\text{th trial}) = q^{k-1} p, \text{ where } k \geq 1.$$

$$\text{Expected value: } \mu = E(X) = \frac{1}{p} \quad \text{Variance: } \sigma^2 = V(X) = \frac{q}{p^2}$$

5. *Negative Binomial.* Bernoulli trials repeated until the k th success, with probability of success p and of failure $q = 1 - p$. X counts the number of trials required.

$$\text{Probability function: } m(x) = P(k \text{ success on } x\text{th trial}) = \binom{x-1}{k-1} p^k q^{x-k}$$

$$\text{Expected value: } \mu = E(X) = \frac{k}{p} \quad \text{Variance: } \sigma^2 = V(X) = \frac{kq}{p^2}$$

Continuous Distributions

1. *Continuous Uniform.*

$$\text{Density function: } f(t) = \begin{cases} \frac{1}{b-a} & a \leq t \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Expected value: } \mu = E(X) = \frac{a+b}{2} \quad \text{Variance: } \sigma^2 = V(X) = \frac{(b-a)^2}{12}$$

2. *Exponential.*

$$\text{Density function: } f(t) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$\text{Expected value: } \mu = E(X) = \frac{1}{\lambda} \quad \text{Variance: } \sigma^2 = V(X) = \frac{1}{\lambda^2}$$

* With apologies to the creators of *Buckaroo Banzai*. “Remember: No matter where you go, there you are.”

3. *Standard normal.*

Density function: $\varphi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$

Expected value: $\mu = E(X) = 0$ *Variance:* $\sigma^2 = V(X) = 1$

4. *Normal. . . with mean μ and standard deviation σ .*

Density function: $f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(t-\mu)^2/2\sigma^2}$

Expected value: $E(X) = \mu$ *Variance:* $V(X) = \sigma^2$