Mathematics 1550H – Probability I: Introduction to Probability
Trent University, Summer 2020 (S62)
Solution to Assignment #1
Is it really random?

Consider the following three sequences of one hundred heads and tails each:

Sequence #1:

```
HTTHHHTTHTTHHTTHTTHTTTTHHHTTTHHTHTTHHTTTTHTTTTTHHT
HTHTHHHTTTHTHHTHHTHHTHTTHTTHTTHHHHHHHHTHHTTTTTHHHHH
```

Sequence #2:

```
HHTHTTTHTTHTHTHHHTTTHTTHHHTHTTTHTHTHHHTHTHTTTHTTTTTHHHT
THTTTHTTHHHTHTHHTTTHHHTTTHTHTTHTHTHTTTHTTHHTHHTTTH
```

Sequence #3:

```
THTHTHHTHHHTTTHHHTTTTTTTTTHHHTTTHHHHTHTHHHTHTTTHTHH
THTTHHHTTTTTTTTHHTHHHHHTHHHHHHHHTHHHHHTTHHHHTTHTTHT
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At least one of these sequences was generated by actually tossing a quarter one hundred times, and at least one was generated by a human sitting at a computer and hitting the "H" and "T" keys one hundred times between them and trying (possibly not very hard) to make it seem random.

1. Try to figure out which of these sequences were generated by actually tossing a coin and which were generated a human trying to be random. Give your reasoning! *[10]*

SOLUTION. The first clue comes from counting the number of heads and tails in each sequence. The first sequence has 49 heads and 51 tails, the second has 45 heads and 55 tails, and the third has 54 heads and 46 tails. This suggests that the first sequence is different, but this difference is not all that large.

The second clue comes from considering long runs of either all heads or all tails in each sequence. The longest run in the first sequence is one of length 7 (of heads), with no runs of length 6 and several runs of length 5. The longest run in the second sequence is is one of length 4 (of tails), with a number of runs of lemgth 3. Finally, the longest runs in the third sequence are of length 9 (of tails) and 8 (of heads), with the next longest being a run of length 7 (of tails) and two runs of length 5 (both of heads). The third sequence has the most long runs (and the longest, too), the second sequence has one long one and some on the short end of long, while the first sequence has no long runs.

Tossing a coin for yourself one hundred times and having a human generate random-seeming sequence by hand will probably, especially if you're patient enough to repeat the experiment several times, reveal to you that genuine sequences don't get really close to a 50-50 split of heads and tails as often as humans do and are more likely to contain some longh runs, and more of them too. This suggests that the third sequence was obtained by tossing a coin, while the other two were generated by humans trying to seem random and partly succeeding: an uneven split of heads and tails in the second sequence and at least one properly long run in the first sequence.

This is indeed the case. Sequence #1 was generated at a keyboard by the instructor's son (Thanks, Max!), sequence #2 was generated at a keyboard by the instructor, while sequence #3 was generated by the instructor tossing a coin 100 times. ∎