

Mathematics 1550H – Probability I: Introduction to Probability
 TRENT UNIVERSITY, Summer 2020 (S62)
Solutions to Quiz #2

Consider the following experiment:

Toss a fair coin twice. If a head comes up on one or both tosses, the experiment ends. Otherwise, proceed to:

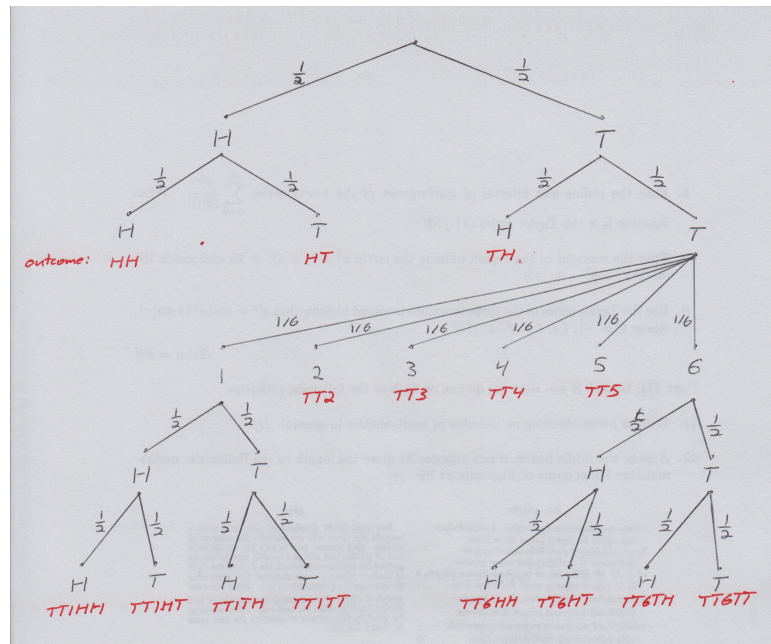
Roll a fair standard six-sided die. If it comes up with 2, 3, 4, or 5, the experiment ends. Otherwise, proceed to:

Toss a fair coin twice. Whatever comes up on the two tosses the experiment ends.

The *score* of the experiment is the number of heads that came up in the course of the experiment plus the number on the die roll, if one occurred.

1. Draw the complete tree diagram for this experiment. [2]

SOLUTION. Here it is:



□

2. What is the probability that every coin toss in the experiment came up the same way? [0.5]

SOLUTION. This is the event $A = \{HH, TT2, TT3, TT4, TT5, TT1TT, TT6TT\}$. Its

probability, computed with the help of the tree diagram, is:

$$\begin{aligned}
 P(A) &= m(HH) + m(TT2) + m(TT3) + m(TT4) + m(TT5) + m(TT1TT) + m(TT6TT) \\
 &= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{2} \\
 &\quad + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{2} \\
 &= \frac{24}{96} + \frac{4}{96} + \frac{4}{96} + \frac{4}{96} + \frac{4}{96} + \frac{1}{96} + \frac{1}{96} = \frac{42}{96} = \frac{7}{16} = 0.4375 \quad \square
 \end{aligned}$$

3. What is the probability that the score of the experiment is 3? [0.5]

SOLUTION. This is the event $B = \{TT3, TT1HH\}$, which has probability, computed with the help of the tree diagram:

$$P(B) = m(TT3) + m(TT1HH) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{4}{96} + \frac{1}{96} = \frac{5}{96} \approx 0.05208 \quad \square$$

4. What is the probability that the score of the experiment is an even number? [2]

SOLUTION. This is the event $C = \{HH, TT2, TT4, TT1HT, TT1TH, TT6HH, TT6TT\}$, which has probability, computed with the help of the tree diagram:

$$\begin{aligned}
 P(C) &= m(HH) + m(TT2) + m(TT4) + m(TT1HT) + m(TT1TH) + m(TT6HH) \\
 &\quad + m(TT6TT) \\
 &= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{2} \\
 &\quad + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{2} \\
 &= \frac{24}{96} + \frac{4}{96} + \frac{4}{96} + \frac{1}{96} + \frac{1}{96} + \frac{1}{96} + \frac{1}{96} = \frac{36}{96} = \frac{3}{8} = 0.375 \quad \square
 \end{aligned}$$