Assignment #5
Random Walks the Plane
Due on Friday, 24 July.

Please submit your solutions using Blackboard's assignment module. If that fails, please email your solutions to the instructor (`sbilaniuk@trentu.ca`). Scans or photos of handwritten solutions are perfectly acceptable, so long as they are legible and in some common format. (Combined into a single pdf, for preference.)

Random decides to take a walk in the Cartesian plane, starting at the origin $(0,0)$. Random being random, the next step is always decided by simultaneously rolling two fair standard dice, a red one and a blue one. If the blue one comes up 1 or 2, Random moves left one unit; if it comes up 3 or 4 Random doesn't move to either side; if it comes up 5 or 6, Random moves right one unit. At the same time, if the red one comes up 1 or 2, Random moves down by 1; if it comes up 3 or 4, Random doesn't move up or down; if it comes up 5 or 6, Random moves up one unit. For example, if Random is at the point $(3,1)$ and rolls 1 on the blue die and 4 on the red die, Random will step to the point $(3-1, 1+0) = (2,1)$.

Let (X_n, Y_n) denote Random's position after n steps. Of course, $(X_0, Y_0) = (0,0)$, but after that chance rules.

1. Find the expected values and variances of X_n and Y_n. *[5]*

2. Explain why it is hard to compute the expected value and variance of the distance that Random is from the origin after n steps. Can you suggest reasonable alternatives to the expected value and/or variance of distance that are easier to compute? *[5]*