Mathematics 1550H – Probability I: Introduction to Probability

TRENT UNIVERSITY, Summer 2020 (S62)

Assignment #2

Cunning Experimental Design

Due on Friday, 3 July.

In each of the following problems you are given a tool and asked to design an experiment using that tool that meets certain requirements.

1. You are given a coin, whether fair or not you do not know, and asked to use it to randomly generate a yes or no answer, with the stipulation that the probability of a yes should be equal to the probability of a no. How can you do the job without having to determine the possible bias of the given coin? Explain why your method works. *[3]*

2. You are given a fair coin and asked to use to to randomly generate a yes or no answer, with the stipulation that the probability of a yes should be exactly $\frac{8}{13}$ and the probability of a no should be exactly $\frac{5}{13}$. How can you do the job? Explain why your method works. *[3]*

3. You are given a fair coin and asked to use to to randomly generate a yes or no answer, with the stipulation that the probability of a yes should be exactly $\frac{1}{\sqrt{2}}$ and the probability of a no should be exactly $1 - \frac{1}{\sqrt{2}}$. How can you do the job? Explain why your method works. *[4]*

NOTE: **3** is different from **2** in that $\frac{1}{\sqrt{2}}$ is an irrational number, and so cannot be expressed precisely as a ratio of integers. This means that the methods most people find to solve **2** do not really work for **3**.

Hint: To solve **3** it helps to think of $\frac{1}{\sqrt{2}}$ in terms of its decimal or, even better, it's binary expansion.