A Few Common Distributions

(Chapter 5 - or at least the parts we'll be using. A large part of this summarizes things we've already seen.)

Discrete Distributions

1. Discrete uniform distribution

A finite sample space with n elements. Each $\omega \in \Omega$ gets an equal weight

ie $m(\omega) = \frac{1}{n}$

2. Bernoulli trial

$\Omega = \{\text{success}, \text{failure}\}$

The probability of success, $m(\text{success}) = p$,

& — " — failure, $m(\text{failure}) = q = 1-p$

To be interesting, we need $0<p<1$.

3. Binomial distribution

Perform n Bernoulli trials with (independent of one another)
probability of success p &
probability of failure $q = 1-p$.

X counts the number of successes in n trials

$$P(X=k) = m(k) = \binom{n}{k} p^k q^{n-k}$$

4. Geometric distribution

Repeat Bernoulli trials with (independent!)
probability of success p
& — " — failure q
until the first success occurs.

X counts how many trials you performed to get the first success

$$P(X=k) = P(\text{first success occurred on the } k^{th} \text{ trial})$$
$$m(k) = q^{k-1} p$$

(note: geometric series happen here

$a + ar + ar^2 + \cdots + ar^n$
$= a \dfrac{1-r^{n+1}}{1-r}$

As long as $|r| < 1$,
$a + ar + ar^2 + \cdots$
$= \dfrac{a}{1-r}$.)

5. Negative Binomial

Repeat Bernoulli trials (independent) until the k^{th} success

(prob. of success $= p$
— " — failure $= q = 1-p$)

X counts the number of trials required for the k^{th} success to occur.

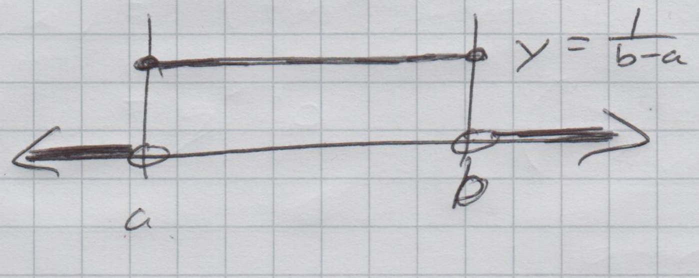
$$m(x) = P(X=x) = \binom{x-1}{k-1} p^k q^{x-k}$$

Continuous Distributions

1. Continuous uniform distribution

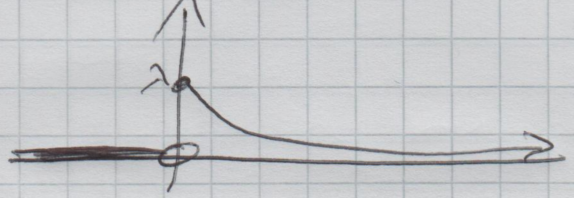
Probability distributed equally over $[a,b]$.

Density function: $f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & x < a \text{ or } x > b \end{cases}$



2. Exponential distribution

Density function: $f(t) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}$

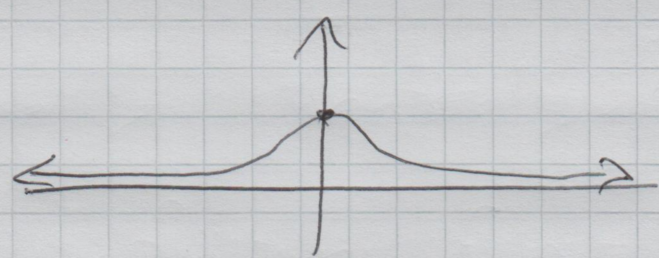


λ is a constant > 0

3. Standard Normal distribution

Density function: $\underset{\text{phi } \phi}{g}(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$

(The classic "bell curve"...)



Z – the random variable

$$P(2 \leq Z \leq 4) = \int_2^4 \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt$$

"has no antiderivative in elementary terms"

How do we compute with it?

We'll use a Standard Normal Table to approximately compute it.

eg $P(2 \leq Z \leq 4)$

$$\doteq P(Z \leq 4) - P(Z \leq 2)$$

is close enough to 1 to be indistinguishable to the naked.

$$\hat{=}\ 1 - P(Z \leq 2.00)$$

$$\hat{=}\ 1 - 0.9772$$

$$\hat{=}\ 0.0228$$

4. Normal distribution with mean μ & variance σ^2 $(\sigma > 0)$

Density function:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(t-\mu)^2/2\sigma^2}$$

Standard normal has $\mu = 0$ & $\sigma = 1$.

<u>Fact:</u> If X has a normal distribution with mean $\overset{\text{"mu"}}{\mu}$ and variance $\overset{\text{"sigma"}}{\sigma}^2$ then $Z = \dfrac{X-\mu}{\sigma}$ has a standard normal distribution.

This means that we can use standard normal tables to compute probabilities for other ~~Ph~~ normal distributions:

$$P(a \leq X \leq b) = P\left(\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma}\right).$$