MATH 1550H Test
Monday, 10 July
Time: 50 minutes

Instructions
- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet.

1. Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

   a. Determine whether $f(x) = \begin{cases} \frac{1}{2}e^x & x \leq 0 \\ \frac{1}{2}e^{-x} & x \geq 0 \end{cases}$ is a valid probability density.

   b. A hand of five cards is drawn randomly, one at a time (so order matters) and without replacement, from a standard 52-card deck. What is the probability that the hand includes exactly three $\heartsuit$s, given that the first card drawn was a $\spadesuit$?

   c. A fair coin is tossed until it comes up heads for the second time. What is the probability that at least four tosses will be required?

2. Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

   a. Suppose that A and B are events in some sample space, with $P(A) = P(B) = \frac{2}{3}$. What is the range of possible values of $P(A|B)$?

   b. The continuous random variable X has the density function $g(x) = \begin{cases} \frac{2}{9}x & 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$.

   Compute the probability that $1 \leq X$, given that $X \leq 2$.

   c. A fair standard six-sided die is rolled once. If it comes up with an odd number, it is rolled just one more time; if it comes up with an even number, it is not rolled again. Compute the probability that the last roll made came up with 1 or 5.

3. Do *one (1)* of **a** or **b**. *[10]*

   a. The continuous random variable W has the density function $h(x) = \begin{cases} xe^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$.

   Compute $P(W \geq 1)$.

   b. A fair coin is tossed until it comes up heads for the third time. Let the random variable Y count the total number of tosses that occur in this experiment. Find the probability funtion of Y and compute $P(Y \leq 5)$.

[Total = 30]