

## Mathematics 1550H – Introduction to probability

TRENT UNIVERSITY, Summer 2017

### Solutions to Assignment #3

#### The Mathematical Inquisition Strikes Again!

In the Kafka-esque dystopian future in which mathematical scientists have taken over the world, you are condemned\* to working out the base 7 expansion of  $\pi$  by hand unless you can correctly answer the following question:

*If you were to pick an answer to this question at random from among the choices below, what is the probability that it would be correct?*

- (a) 0.2
- (b)  $1/\pi$
- (c)  $1/5$
- (d) 0.0
- (e)  $4/10$

#### 1. Explain! [5]

SOLUTION. There is no probability that can be properly assigned here. Each of answers (a)–(e) has a  $\frac{1}{5} = 0.2$  chance of being chosen at random. Unfortunately, (a) and (c) both give this number, so one would have a chance of  $\frac{2}{5} = 0.4$  of getting a correct answer, and  $\frac{2}{5} \neq \frac{1}{5}$ . Answer (d),  $\frac{4}{10}$ , is actually equal to  $\frac{2}{5}$ , but only has a  $\frac{1}{5}$  chance of being chosen. The probability of picking a correct answer can be 0 because that is answer (d), which also has a chance of  $\frac{1}{5} \neq 0$  of being chosen. The less said about answer (b), the better ... :-)

The problem is actually a self-referential paradox: the self-reference (“If you were to pick an answer to this question ... ”), combined with the given answers, makes all the possible answers wrong, including probability 0.  $\square$

While serving your sentence of computing the base 7 expansion of  $\pi$  by hand, your appeal goes forward. The court decides it will reduce your sentence to time served if you can answer the following question correctly:

*How many letters are there in the answer to this question?*

#### 2. Well? [5]

SOLUTION. There are several possible answers. First, “four” is a possible answer because the word has 4 letters. Second, “0” is also a possible answer since the digit “0” has no letters. Third, Jennifer Lennick came up with one I haven’t seen before, “exactly ten”, which works because the phrase “exactly ten” uses 10 letters [and a space, which doesn’t count as a letter]. (Thanks, Jennifer! :-) There may well be more possible answers out there ...  $\blacksquare$

Remember: no one expects the Mathematical Inquisition!

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\* For writing that  $\frac{1}{2} + \frac{1}{3} = \frac{1+1}{2+3} = \frac{2}{5}$ . At least you didn’t divide by 0 ... :-)