Mathematics 1550H – Introduction to probability

Trent University, Summer 2017

Solutions to Assignment #1

Ups and Downs

Meredith works on the 13th floor of a 15-floor building. The only elevator moves continuously through floors 1, 2, ... , 15, 14, ... , 2, 1, 2, ... , except that it stops on a floor on which the button has been pressed. Assume that time spent loading and unloading passengers is very small compared to the travelling time. Meredith complains that at 5 p.m., at the end of the working day, the elevator almost always goes up when it stops on the 13th floor floor.

1. What is the explanation for this? Compute the probability that the elevator is going down when Meredith wants to go home at 5 p.m. *[5]*

Solution. We can reasonably assume that the elevator is equally likely to be at any point between the first floor and the fifteenth floor at any point in time. We can also assume that the probability that the elevator is exactly on the thirteenth floor when Meredith arrives is negligible. This gives a probability of $\frac{2}{14} = \frac{1}{7} \approx 0.1429$ that it is above the thirteenth floor (which is when it will go down when it goes by this floor) when Meredith wants to go home. □

2. Now assume that the building has n elevators, which move independently. Compute the probability that the first elevator to arrive on Merediths floor at 5 p.m. is moving up. *[4]*

Solution. Suppose we have $n > 1$ independently moving elevators. [Independence means that knowing where one elevator is and whether it is moving up or down tells you nothing about where any other elevator is or which way it's going.] Call the *unbiased portion* the part of each elevator's route up from the ninth floor to the top and then down to the thirteenth floor. Any elevator at a random spot of the unbiased portion is equally likely to go up or down when it goes by the thirteenthth floor. Moreover, if there is at least one elevator in the unbiased portion, all elevators out of it do not matter. However, if no elevator is in the unbiased portion, then the first one to reach the thirteenth floor goes up. Therefore the probability that the first elevator to stop at the thirteenth floor is going down equals $\frac{1}{2}\left(1 - \left(\frac{10}{14}\right)^n\right)$ [Why, exactly? Think it through.] and the probability that it is going up is $1 - \frac{1}{2}\left(1 - \left(\frac{10}{14}\right)^n\right) = \frac{1}{2} + \frac{1}{2}\left(\frac{10}{14}\right)^n$. For $n = 2$ this is about 0.7551. □

3. Is there a number n of elevators (moving independently) such that Meredith has at least an even chance of catching a downward-moving elevator at 5 p.m.? If so, what is it? If not, why not? *[1]*

Solution. Since $\left(\frac{10}{14}\right)^n > 0$ for all n, $1 - \left(\frac{10}{14}\right)^n < 1$ for all n, and hence there is no n such that $\frac{1}{2}\left(1 - \left(\frac{10}{14}\right)^n\right) = \frac{1}{2}$. This means that Meredith does have an even chance or better of catching a downward-moving elevator at 5 p.m., no matter how many elevators the building has. ■