Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Summer 2017
Quizzes

Quiz #1. Wednesday, 21 June, 2017. *[10 minutes]*

Consider the following experiment. A fair coin is tossed twice. If it comes up heads on the second toss, the experiment ends; if it comes up tails on the second toss, the coin is tossed just one more time, ending the experiment.

1. List all the outcomes in the sample space Ω for this experiment. *[1.5]*
2. What is the probability distribution function for this experiment? *[1.5]*
3. Let E be the event that an odd number of heads came up in the experiment. Compute the probability of E. *[2]*

Quiz#2. Monday, 26 June, 2017. *[12 minutes]*

Consider the following experiment. A fair coin is tossed once, and then tossed repeatedly until the face that came up on the first toss comes up again.

1. What is the sample space for this experiment? *[1.5]*
2. What is the probability distribution function for this experiment? *[1.5]*
3. Let X count the total number of tosses made in the experiment. What is the probability that $X \geq 5$? *[2]*

Quiz #3. Wednesday, 28 June, 2017. *[10 minutes]*

1. A hand of five cards is drawn simultaneously [so without order or replacement] from a well-shuffled standard deck. What is the probability that the hand has "two pairs" [*i.e.* two cards of one kind, two cards of a different kind, and one card of yet another kind]? *[5]*

Quiz #4. Wednesday, 5 July, 2017. *[15 minutes]*

The continuous random variable X has $f(x) = \begin{cases} 3x^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$ as its probability density function. Let A be the event that $X \leq \frac{1}{2}$ and let B be the event that $\frac{1}{4} \leq X$.

1. Compute $P(A|B)$, the conditional probability of A given B. *[5]*

Quiz #5. Wednesday, 12 July, 2017. *[10 minutes]*

Do *one* (1) of the following questions.

1. The continuous random variable X has $f(x) = \begin{cases} 1 - \frac{1}{2}x & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$ as its probability density function. Find the expected value, $E(X)$, of X. *[5]*
2. A fair coin is tossed until it comes up heads or until three tosses have been made, whichever comes first. The random variable Y counts how many tosses came up tails in the experiment. Find the expected value, $E(Y)$, of Y. *[5]*

Quiz #6. Monday, 17 July, 2017. *[15 minutes]*

Do *one* (1) of the following questions.

1. The continuous random variable X has $g(x) = \begin{cases} \frac{3}{4}\left(1 - x^2\right) & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$ as its probability density function. Find the variance, $V(X)$, of X. *[5]*
2. A fair four-sided die with faces numbered 1 through 4 is rolled twice. The discrete random variable W records the sum of the faces. Find the the variance, $V(W)$, of W. *[5]*

Quiz #7. Wednesday, 19 July, 2017. *[15 minutes]*

The continuous random variables X_1, X_2, X_3, and X_4 are independently and identically distributed with a normal distribution that has expected value $\mu = 3$ and standard deviation $\sigma = 4$. Let $\bar{X} = \dfrac{X_1 + X_2 + X_3 + X_4}{4}$; then $\bar{X}$ also has a normal distribution with expected value $\mu = 3$ (albeit it has a different variance and standard deviation).

1. Compute $P(-2 \leq X_1 \leq 7)$. *[2]*
2. Compute $P\left(-2 \leq \bar{X} \leq 7\right)$. *[3]*

Quiz #8. Monday, 24 July, 2017. *[15 minutes]*

Suppose Z has a standard normal distribution.

1. Compute $P(|Z| > 2)$ using a standard normal cumulative probability table. *[1]*
2. Find an upper bound for $P(|Z| > 2)$ using Chebyshev's Inequality. *[2]*
3. Extract the smallest upper bound for $P(Z > 2)$ that you can from your calculation for question 2. *[2]*

Quiz #9. Wednesday, 26 July, 2017. *[20 minutes]*

Suppose the discrete random variables X and Y are jointly distributed according to the following table:

$X \backslash Y$	1	2	3
−1	0.1	0.1	0.2
0	0.1	0	0.1
1	0.2	0.1	0.1

1. Compute the expected values $E(X)$ and $E(Y)$, variances $V(X)$ and $V(Y)$, and covariance $\text{Cov}(X, Y)$ of X and Y. *[4]*
2. Let $U = X - 2Y$. Compute $E(U)$ and $V(U)$. *[1]*