Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Summer 2017

FINAL EXAMINATION
Saturday, 29 July, 2017

Spatio-temporal locus: 14:00–17:00 in FPHL 117 *Inflicted by* Стефан Біланюк.

Instructions: Do both of parts **Card** and **Coin**, and, if you wish, part **Die**. Show all your work and simplify answers as much as practicable. *If in doubt about something,* **ask!**

Aids: Calculator; $8.5'' \times 11''$ or A4 aid sheet; standard normal table; one brain (caffeine optional).

Part Card. Do all of **1**–**5**. *[Subtotal = 68/100]*

1. A hand of five cards is drawn randomly, simultaneously and without replacement, from a standard 52-card deck.

a. What is the probability that all the cards in the hand are ♢s and/or ♣s? *[5]*

b. What is the probability that the hand is a *full house*, consisting of three of one kind and two of another kind? *[5]*

c. What is the probability that the hand would be counted in both **a** and **b**? *[5]*

2. Let W be a continuous random variable with the following probability density function:

$$g(w) = \begin{cases} w^{-2} & w \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Verify that $g(w)$ is indeed a probability density function. *[8]*

b. Compute the probability that $W \geq 4$, given that $W \geq 2$. *[7]*

c. Find the expected value, $E(W)$, of W. *[5]*

3. A fair coin is tossed, and then tossed repeatedly until it comes up with a face different from the one that came up on the first toss.

a. Draw the tree diagram for this experiment. *[3]*

b. What are the sample space and probability function for this experiment? *[5]*

c. Let the random variable X count the total number of tosses that occur in the experiment. Find the expected value $E(X)$ and variance $V(X)$ of X. *[7]*

4. The continuous random variable Y has an exponential distribution with variance $V(Y) = 4$. What is the probability density function of Y? *[5]*

5. Suppose X is a continuous random variable that has a normal distribution with expected value $\mu = -2$ and standard deviation $\sigma = 5$.

a. Compute $P(1 \leq X \leq 5)$ with the help of a standard normal table. *[6]*

b. Use Chebyshev's Inequality to get as small an upper bound for $P(X \geq 8)$ as you can. *[7]*

[Parts **Coin** *and* **Die** *are on page 2.]*

Part Coin. Do any *two* (2) of **6**–**9**. *[Subtotal = 32/100]*

6. You are given two bowls and 180 marbles, 100 of them black, 50 of them white, and 30 of them green. A blindfolded assistant will select a bowl at random, and then select a marble at random from that bowl. How should you distribute the marbles between the two bowls to maximize the probability of the assistant selecting a white marble? Provide as complete an explanation as you can. *[16]*

7. A fair coin is tossed ten times. The random variable X counts how many pairs of consecutive tosses had the same face come up.

a. What are the possible values and probability function for X? *[8]*

b. Find the expected value $E(X)$ and variance $V(X)$ of X. *[8]*

8. Suppose that $g(x) = \begin{cases} 0 & x < -1 \\ x+1 & -1 \le x < 0 \\ \frac{1}{2}e^{-x} & x \ge 0 \end{cases}$ is the probability density function of the continuous random variable X.

a. Verify that $g(x)$ is indeed a probability density function. *[6]*

b. Compute the expected value $E(X)$ and variance of $V(X)$ of X. *[10]*

9. The discrete random variables X and Y are jointly distributed according to the given table:

$X \backslash Y$	1	2	3
-1	0.1	0.1	0.2
1	0.1	0	0.1
3	0.2	0.1	0.1

a. Compute the expected values $E(X)$ and $E(Y)$, variances $V(X)$ and $V(Y)$, and covariance $\mathrm{Cov}(X, Y)$ of X and Y. *[12]*

b. Let $U = -X - 2Y$. Compute $E(U)$ and $V(U)$. *[4]*

[Total = 100]

Part Die. Bonus!

• . In series of games numbered 1, 2, 3, ... , the winning number in the nth game is randomly chosen from the set $\{1, 2, \ldots, n+2\}$. Kosh Naranek bets on the number 1 in each game and intends to keep playing until (s)he wins once. What is the probability that Kosh will have to play forever? *[1]*

⁘. Write an original little poem about probability or mathematics in general. *[1]*

[Part **Card** *is on page 1.]*

Have a great August!